[This question paper contains 4 printed pages.]

1058 *Your Roll No.* ....

B.Sc. (Hons.) / III **C**

STATISTICS – Paper XXI

C–223 : (Linear Models)

(Admissions of 1999 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **Four** *questions in all, selecting* **two** *questions from each Section.*

SECTION I

(a) Suppose x_i, y_i, z_i, $i = 1, 2, \ldots, n$ are 3n independent observations with common variance σ^2 and expectation given by $E(x_i) = \theta_1$, $E(y_i) = \theta_2$, and $E(z_i) = \theta_1 - \theta_2$, $i = 1, 2, \ldots, n$. Find the BLUEs of θ_1, θ_2 and compute the residual sum of squares. Also find blue of $\theta_1 + \theta_2$ and its variance.

P.T.O.

(b) Consider the simple linear regression model $Y = \beta_0 + \beta_1 x + \epsilon$ with usual assumptions. Obtain unbiased point estimator and interval estimator of the mean response for a particular value of the regressor variable. (6,3½)

2. (a) Let $Y'AY$ be Quadratic Form in $y_1, \ldots, y_n$ where $y_i \sim N(0,1)$, $i=1,\ldots,n$. Prove that A is an idempotent matrix of rank k if $Y'AY$ is distributed as χ^2 with k df.

(b) If $Y \sim N_p(\mu, \Sigma)$. Obtain the distribution of

$$(Y - \mu)' \sum^{-1} (Y - \mu).$$

(c) Let $Y \sim N_3(0, \Sigma)$ where

$$\sum = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$

If $A = \begin{bmatrix} 1 & -3 & -8 \\ -3 & 2 & -6 \\ -8 & -6 & 3 \end{bmatrix}$ Find expected value of $Y'AY$. (4½,2½,2½)

3. What are the basic differences between fixed effect model and random effect model ? Derive the analysis

of variance of two way classified data with m observations per cell under random effect model. (9½)

SECTION II

4. (a) Write short notes on :

   (i) Estimable Functions

   (ii) Orthogonal Columns in X matrix

   (iii) Coefficient of determination

   (b) Suppose we postulate the model $E(y) = \beta_1 x$ but the true model is $E(y) = \beta_0 + \beta_1 x$. Obtain the bias in estimate of β_1. (6,3½)

5. (a) Define polynomial regression models. Explain the role of orthogonal polynomials in fitting polynomial models in one variable.

   (b) Consider the multiple linear regression model. "Which specific regressors seem important ?" How will you address this question ? (4,5½)

6. (a) Consider the model $Y=X\beta+\varepsilon$ where $E(Y) = X\beta$, $CoV(Y) = \sigma^2 I$ and X is $n \times p$ of rank $k < p \leq n$. Obtain an unbiased estimator of σ^2.

(b) Consider the model $E(y_{ij}) = a_i + b_j$, $i = 1,2$; $j = 1,2$ with usual assumptions. Obtain the BLUE of $a_1 + b_1$. (4,5½)