This question paper contains 4+2 printed pages]

Your Roll No. ....

1061

B.Sc.(Hons.)/III **C**

STATISTICS—Paper XXIV

(Stochastic Processes)

(Admissions of 1999 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Answer *four* questions in all,

selecting *two* questions from each Section.

Section I

1. (*a*) Let X_i, $i = 1, 2, \ldots$ be i. i. d random variables with $P[X_i = k] = p_k$ and p.g.f. $P(s) = \sum p_k s^k$. Define $S_N = X_1 + X_2 + \ldots + X_N$, where N is a random variable independent of X_i's. Let the probability function

P.T.O.

of N be $P[N = n] = g_n$ and the p.g.f. be $G(s) = \sum g_n s^n$.

Show that the p.g.f. of S_N is given by the compound function $G(P(s))$. Also find $E(S_N)$.

(b) Let X have a truncated Poisson distribution with zero class missing i.e. :

$$p_k = P[X = k] = \frac{(e^a - 1)^{-1} a^k}{k!}, \; k = 1,2,3\ldots\ldots$$

Find :

(*i*) p.g.f. of X

(*ii*) E [X] and

(*iii*) V [X].

(c) Consider the process :

$$X(t) = A\cos\omega t + B\sin\omega t,$$

where A, B are uncorrelated r.v.'s each with mean 0 and variance 1 and ω is a positive constant. Is the process covariance stationary ? 3½,3,3

2. (a) Consider the Markov chain with the transition matrix :

		X_n			
		1	2	3	4
X_{n-1}	1	1/2	1/4	1/4	0
	2	0	0	1	0
	3	1/3	0	1/3	1/3
	4	0	0	0	1

(i) Is the chain irreducible ?

(ii) Find the nature of the states 2 and 4.

(b) Prove that the generating functions of $\{U_n\}$ and $\{f_n\}$ are related by :

$$U(s) = \frac{1}{1-F(s)}, \text{ where}$$

$U_n = p$ [E occurs at the nth trial],

$f_n = p$ [E occurs for the first time at nth trial].

Further state when E will be called :

(i) Transient,

(ii) Persistent. 5½,4

3. (*a*) Let the distribution of number of off-springs be :

$$p_k = bc^{k-1}, \quad k = 1, 2, \ldots\ldots, \quad 0 < b, c, b + c < 1$$

$$\text{and} \quad p_0 = 1 - \sum_{k=1}^{\infty} p_k$$

Find P(s) (p.g.f. of off-spring distribution) and the probability of ultimate extinction.

(*b*) Prove that in a G.W. Branching process :

$$P_n(s) = P_{n-1}(P(s)) \quad \text{and} \quad P_n(s) = P((P_{n-1}(s))$$

where $P_n(s) = \sum_k P[X_n = k]\, s^k$ and

$$P(s) = \sum_k P[Z_r = k]\, s^k .$$ 4,5½

Section II

4. (*a*) Set up the differential equations for $p_n(t)$ in the case of pure birth process with rate λ_n. Hence find $p_n(t)$ and p.g.f. for the *Yule Furry process*, starting with one individual. Identify the distribution.

(*b*) Show that for a Poisson process $\{N(t)\}$, as $t \to \infty$

$$P\left\{\left|\frac{N(t)}{t} - \lambda\right| \geq \varepsilon\right\} \to 0$$

where $\varepsilon > 0$, is a preassigned number. 5½,4

5. (*a*) State the classical ruin problem. Find an expression for the expected duration of the game which is expected to be finite.

(*b*) Obtain the optimal replacement policy in case of items whose maintenance and repair cost increases with time ignoring changes in the value of money. 4½,5

6. (*a*) For a (M/M/c) : (∞/FCFS) model, derive the expression for :

(*i*) Probability that there are *n* customers in the system.

(*ii*) The average number of customers in the system.

(*iii*) The average queue length.

(*b*) A. T.V. repairman finds that the time spent on his jobs has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they came in, and if the arrival of sets is approximately Poisson with an average rate of 10 per 8-hour day, then :

(*i*) Identify the model.

(*ii*) What is the repairman's expected idle time each day ?

(*iii*) How many jobs are ahead of the average set just brought in ? 5,4½