This question paper contains 4+2 printed pages]

Your Roll No. .....

1062

B.Sc.(Hons.)/III **C**

STATISTICS—Paper XXV

(Econometrics)

(Admissions of 1999 and onwards)

Time : 2 *Hours* *Maximum Marks* : 38

(Write your Roll No on the top immediately on receipt of this question paper.)

Answer *four* questions in all,

selecting *two* from each Section.

Section I

1. (*a*) In the general linear regression model $Y = X\beta + u$, derive the test for the hypothesis $H_0 : c'\beta = r$, where c is an appropriately specified vector of constants and r is a known constant.

(*b*) What do you mean by lag models ? Discuss adaptive expectation model and its features in detail.

P.T.O.

(*c*) Consider the following correlation matrix :

$$R = \begin{array}{c} \\ X_2 \\ X_3 \\ \vdots \\ X_k \end{array} \begin{array}{c} \begin{array}{cccc} X_2 & X_3 & \cdots & X_k \end{array} \\ \begin{bmatrix} 1 & r_{23} & \cdots & r_{2k} \\ r_{32} & 1 & \cdots & r_{3k} \\ \vdots & \vdots & \cdots & \vdots \\ r_{k2} & r_{k3} & \cdots & 1 \end{bmatrix} \end{array}$$

How would you find out from the correlation matrix whether :

(*i*) there is perfect multi-collinearity;

(*ii*) there is less than perfect multi-collinearity;

(*iii*) the X's are uncorrelated. 3,4½,2

2. (*a*) Given the model $Y_t = \alpha + \beta X_t + u_t$, where u_t's follow the first order autoregressive scheme $u_t = \rho u_{t-1} + \varepsilon_t$, ε_t satisfies the assumptions of the classical linear regression model :

(*i*) Show that $\text{Var}(u_t) = \sigma^2/(1 - \rho^2)$, where σ^2 is the variance of ε_t.

(*ii*) What is the covariance between u_t and u_{t-1} ? Between u_t and u_{t-2} ? Generalize your results.

(*iii*) Write the variance-covariance matrix of u's. 'In the presence of autocorrelation, the OLS estimators are biased but efficient', the given statement is true or false, justify your answer.

(*b*) Describe in detail at least *four* remedial measures that address the problem of multicollinearity in a given model. 6,3½

3. (*a*) Explain the procedure of exponential smoothing. Show that :

(*i*) the same procedure can be obtained by minimising weighted squared errors;

(*ii*) the smoothed statistic S_T is a weighted average of all the past observations; and

(*iii*) exponential smoothing process yields an unbiased estimate of the true process average b.

P.T.O.

(*b*) Discuss Glejser's test for testing the presence of heteroscedasticity in detail along with its merits and demerits. 5½,4

Section II

4. (*a*) What is constraint utility maximization ? Derive the necessary and sufficient condition for U to be maximum, using differentials, where $U = f(x_1, x_2)$, is the utility function and $Y_0 = p_1x_1 + p_2x_2$ is the budget constraint.

(*b*) The utility function of the consumer is given by $U = x_1x_2^2 - 10x_1$ where x_1 and x_2 are the quantities of two commodities consumed. Find the optimal values of x_1 and x_2 if his income is Rs. 116 and product prices are Rs. 2 and Rs. 8, respectively.

(*c*) For the demand curve $p = (10 - 2x)(20 - x^2)$, find the price elasticity of demand at $x = 4$. 3½,3,3

5. (*a*) What is Engel's law and Engel's curve ? How will you determine it on the basis of family budget data ?

(*b*) Define isoquants and isocost curves. Determine the maximum profit if the production function is :

$$q = \frac{1}{16}[65 - 2(L-5)^2 - 4(K-4)^2]$$

and the prices of inputs L and K are Rs. 8 and Rs. 4 respectively. Also the price of output is Rs. 32. 4,5½

6. (*a*) Define marginal rate of substitution and elasticity of substitution. Show that the elasticity of substitution of the production function :

$$\sqrt{2Hab - Aa^2 - Bb^2}$$

is $$\sigma = \frac{Hx^2}{(H^2 - AB)ab} - 1.$$

(*b*) A monopolist firm produces two types of chocolates X_1 and X_2 at a constant average costs of Rs. 2.50 and Rs. 3.00 per kg. respectively. If p_1 and p_2 are prices charged per kg and market demands are given by :

$x_1 = 5(p_2 - p_1)$ and $x_2 = 32 + 5p_1 - 10p_2$,

obtain the levels at which prices will be fixed for the two types of chocolates for maximum joint monopoly revenue. 5½4