[This question paper contains 4 printed pages.]

2035 *Your Roll No.* ....

B.Sc. (Hons.) / III **E**

STATISTICS – Paper XIX

(Statistical Inference – I)

(Admissions of 1999 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **four** *questions in all selecting* **two** *from each Section.*

SECTION I

1. (a) Define a consistent and unbiased estimator. Suppose X has a truncated Poisson distribution with probability mass function :

$$f(x;\theta) = \frac{\exp(-\theta)\theta^{x}}{[1-\exp(-\theta)]x!} \qquad x = 1, 2, 3, \ldots\ldots .$$

Obtain an unbiased estimator of $[1 - \exp(-\theta)]$ based on X.

P.T.O.

(b) State and prove invariance property of consistent estimators. Using this property obtain consistent estimator of $\theta^2 + \theta - \sqrt{\theta}$, when X follows Bernoulli distribution with parameter θ. (4,5½)

2. (a) Let T be an MVU estimator for $\gamma(\theta)$ and T_1 and T_2 be two other unbiased estimators of $\gamma(\theta)$, with efficiencies e_1 and e_2 respectively. If ρ_0 is the correlation coefficient between T_1 and T_2 then show that :

$$(e_1e_2)^{1/2} - \{(1-e_1)(1-e_2)\}^{1/2} \le \rho_0 \le (e_1e_2)^{1/2} + \{(1-e_1)(1-e_2)\}^{1/2}$$

(b) Given $f(x,\theta) = \frac{1}{\theta}, \ 0 < x < \theta, \ \theta > 0$

compute the reciprocal of $n \ E\left\{\left[\frac{\partial \log f(x,\theta)}{\partial \theta}\right]^2\right\}$ and compare this with the variance of $(n+1)Y_n/n$, where Y_n is the largest observation of a random sample of size n from this distribution. Comment on the result. (5,4½)

3. (a) X has a binomial distribution of index n and parameter θ and the prior probability density function of θ is U[0, 1]. Obtain Bayes estimate and Bayes risk if loss function is $L(\theta,a) = (\theta-a)^2$.

(b) Describe the procedure of obtaining estimates by the method of minimum Chi-square. (4½,5)

SECTION II

4. (a) State and prove Fisher – Neyman criterion for the existence of a sufficient statistic.

(b) X_1, X_2 is a random sample of size 2 from a distribution having probability density function

$$f(x, \theta) = \frac{1}{\theta} e^{-x/\theta}, \qquad 0 < x < \infty,$$

Show that $Y_1 = X_1 + X_2$ is sufficient for θ and find the joint probability density function of Y_1 and $Y_2 = X_2$.

Also show that Y_2 is unbiased for θ with variance θ^2. Find $E(Y_2|y_1) = \phi(y_1)$ and compare the variances of $\phi(y_1)$ and Y_2. (5½,4)

5 (a) Prove that under regularity conditions, with probability approaching unity as n tends to infinity, likelihood equation $\frac{\partial \log L}{\partial \theta} = 0$, has a solution which converges in probability to the true value θ_0.

(b) If $X_1, X_2, \text{-----}, X_n$ is a random sample from the distribution :

$$f(x,\theta) = \frac{\theta^{k+1} x^k e^{-\theta x}}{\Gamma(k+1)}; \quad x > 0, \theta > 0$$

where k is a known constant, show that the maximum likelihood estimator $\hat{\theta}$ of θ is $\frac{k+1}{\bar{x}}$. Also show that the estimator is biased but consistent and that its asymptotic distribution for large n is $N\left(\theta, \frac{\theta^2}{n(k+1)}\right)$. (5½,4)

6. (a) Explain the difference between point estimation and interval estimation. Obtain $100(1-\alpha)\%$ confidence interval for the population correlation coefficient when the random sample of size n has been drawn from a bivariate normal population.

(b) Describe the method of moments.

Estimate α and β by the method of moments for the distribution :

$$f(x,\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, \quad 0 \le x < \infty$$ (5½,4)