[This question paper contains 4 printed pages.]

2036 *Your Roll No.* ....

B.Sc. (Hons.) / III **E**

STATISTICS – Paper XX

(Statistical Inference II)

(Admissions of 1999 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any **FOUR** *questions by selecting* **TWO** *from each Section.*

SECTION – I

1. (a) Define the following :

   (i) Critical region

   (ii) Composite hypothesis

   (iii) Size of the test

   (b) Let p be the probability of appearance of a head in a single toss of a coin in order to test.

P.T.O.

$H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$, the coin is tossed 5 times. H_0 is rejected if more than 3 heads are obtained. Find the probability of type I error and power of the test. (4½,5)

2. (a) Given a random sample $X_1, X_2, X_3, \ldots\ldots\ldots X_n$ from the distribution with probability density function

$f(x;\theta) = \theta e^{-\theta x}$, $x > 0$, obtain UMP critical region for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta < \theta_0$.

(b) Let $X_1, X_2, \ldots\ldots\ldots X_n$ be a random sample from discrete distribution with probability function f(x) for which x takes non-negative integral values 0, 1, 2, ...

According to H_0: $f(x) = \begin{cases} \frac{e^{-1}}{x!}; & x = 0,1,2, \ldots. \\ 0 \ , & \text{otherwise} \end{cases}$

According to H_1: $f(x) = \begin{cases} \frac{1}{2^{x+1}}; & x = 0,1,2, \ldots. \\ 0 \ , & \text{otherwise} \end{cases}$

Obtain the MP critical region of size α for testing H_0 against H_1. Hence obtain the power of the test for the case $n = 1$ and $k = 1$, where k is the constant such that the size of the critical region is α. (4½,5)

3. Describe likelihood-ratio Test. Mention its properties. Construct likelihood ratio test for testing the equality of means of two normal populations having common unknown variance. (9½)

SECTION – II

4. (a) Develop SPRT for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1 (> \theta_0)$ where θ is the parameter of Poisson distribution. Also obtain its OC and ASN functions.

   (b) Discuss Kolmogorov-Smirnov one sample test. (5,4½)

5. (a) If SPRT of strength (α, β) terminates with probability one then determine the boundary points A and B.

   (b) Develop Wald-Wolfowitz run test for the two sample problem. (4½,5)

6. (a) Define Bayes factor. Let $X_1, X_2, \ldots\ldots X_n$ be random sample from $f(x; \theta)$, where θ is unknown. Obtain Bayes factor for testing :

   (i) Simple H_0 against simple H_1,

   (ii) Composite H_0 against composite H_1.

(b) Let X_1, X_2, ... X_n be random sample from $N(\theta, \varnothing)$, $\varnothing$ being known. If θ follows $N(\theta_0, \psi)$, calculate Bayes factor B in favour of testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$. (5,4½)