

[This question paper contains 4 printed pages.]

2037

Your Roll No.

B.Sc. (Hons.) / III

E

STATISTICS – Paper XXI

(Linear Models)

(Admissions of 1999 and onwards)

Time : 2 Hours

Maximum Marks : 38

*(Write your Roll No. on the top immediately
on receipt of this question paper.)*

*Attempt **FOUR** questions in all
selecting **TWO** from each Section.*

SECTION I

1. (a) Consider the simple linear regression model
 $Y = \beta_0 + \beta_1 x + \epsilon$ with $E(\epsilon) = 0$, $V(\epsilon) = \sigma^2$.
Show that

$$(i) \text{COV}(\hat{\beta}_0, \hat{\beta}_1) = -\bar{x} \frac{\sigma^2}{S_{xx}}$$

$$(ii) \text{COV}(\bar{Y}, \hat{\beta}_1) = 0$$

P.T.O.

$$(iii) E(MSR) = \sigma^2 + \beta_1^2 S_{xx}$$

- (b) Consider the simple linear regression model $Y = \beta_0 + \beta_1 x + \epsilon$ with usual assumptions. Obtain unbiased point estimator and confidence interval for the mean response for a particular value of the regressor variable. (5,4½)
2. Let $Y'AY$ be quadratic form in Y_1, Y_2, \dots, Y_n where $Y_i \sim N(0, 1)$, $i=1, \dots, n$. Prove that A is an idempotent matrix of rank k if and only if $Y'AY$ is distributed as χ^2 with k d.f. (9½)
3. Derive the analysis of covariance for a one way layout with one concomitant variable only. (9½)

SECTION II

4. (a) What do you mean by bias in regression estimate? Suppose the postulated model is $E(y) = \beta_0 + \beta_1 x$ but the true model is actually $E(y) = \beta_0 + \beta_1 x + \beta_2 x^2$, what biases are induced in the least squares estimators of β_0 and β_1 by taking the observations at $x = -2, -1, 0, 1, 2$?
- (b) Describe the general linear model and discuss the special cases arising there from. (5½,4)

5. (a) Write a note on the extra sum of squares method that can be used to test the hypotheses about any subset of regressor variables.
- (b) Four objects A, B, C, D are involved in a weighing experiment. Put together they weighed Y_1 grams. When A & C are put in the left pan of the balance and B & D are put in the right pan, a weight of Y_2 grams were necessary in the right pan for the balance. With A & B in the left pan and C & D in the right pan, Y_3 gram was needed in the right pan. Finally, with A & D in the left pan and B & C in the right pan, Y_4 gram was needed in the right pan. If the observations Y_1, Y_2, Y_3, Y_4 are all subject to uncorrelated errors with common variance σ^2 , obtain the BLUE of the total weight of the four objects and its variance. (5½,4)
6. (a) Write short notes on :
- (i) Multiple linear regression models
 - (ii) No-intercept models
 - (iii) Coefficient of determination

- (b) What is a parametric function ? Derive a necessary and sufficient condition for which parametric function is estimable. (5½,4)