[This question paper contains 4 printed pages.]

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Your Roll No. ....

B.Sc. (Hons.) / III **E**

STATISTICS – Paper XXIV

(Stochastic Processes)

(Admissions of 1999 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **four** *questions in all, selecting* **two** *from each Section.*

SECTION I

1. (a) Find the generating function of $\{F_n\}$, defined by

$$F_n = F_{n-1} + F_{n-2},\ n \geq 2,\ F_0 = 0,\ F_1 = 1$$

(b) Let X_i $i = 1,2,\ldots$ be identically and independently distributed random variables with $P\{X_i = k\} = p_k$ and p.g.f. $P(s) = \Sigma_k\ p_k\ s^k$ for $i=1,2,\ldots$. Let $S_N = X_1 + X_2 + \ldots + X_N$, where N is a random variable independent of X_i's. Let the distribution of N be given by $P\{N = n\} = g_n$ and the p.g.f. of N be $G(s) = \Sigma_n g_n s^n$. Obtain the p.g.f. of S_N and $E[S_N]$.

(c) Consider the process $X(t) = A \cos\omega t + B \sin\omega t$, where A and B are uncorrelated random variables each with mean 0 and variance 1 and ω is a positive constant. Is the process covariance stationary ? (3,3½,3)

2. (a) Suppose that a ball is thrown at random to one of the 3 cells. $X_n (n \geq 1)$ is said to be state $k(=1,2,3)$, if after n throws k cells are occupied. Find the transition probability matrix P and hence P^n.

(b) Divide the interval [0,t] into a large number of intervals n each of size h, where h is small and suppose that in each small interval, Bernoulli trials with probability of success λh are held. Show that the number of successes in an interval of length t is a Poisson process with mean λt. State the assumptions you make. (4,5½)

3. (a) Show that in an irreducible Markov Chain all states are of same type.

(b) If $\{N(t)\}$ is a Poisson process then show that the auto correlation coefficient between N(t) and N(t+s) is $\{t/(t+s)\}^{1/2}$. (5½,4)

SECTION II

4. (a) What is a classical ruin problem ? For a classical ruin problem obtain the expected duration of the game.

   (b) Determine the optimum replacement age of an item whose maintenance cost increases with time and value of money remains static during the period when the time is a continuous variable. (5½,4)

5. (a) Let p_k, k =0,1,2 be the probability that an individual in a generation generates k offsprings. Find the probability of ultimate extinction if $p_0 = 2/3$, $p_1 = 1/6$ and $p_2 = 1/6$.

   (b) Obtain the average queue length for the (M/M/1) : (N/FIFO) queueing model having the finite system capacity N. (4,5½)

6. (a) If the offspring distribution in a G.W. process is geometric $\{q^k p\}$, $k = 0,1,2,\ldots$ then show that

$$P_n(s) = \frac{p\left(q^n - p^n\right) - \left(q^{n-1} - p^{n-1}\right)pqs}{\left(q^{n+1} - p^{n+1}\right) - \left(q^n - p^n\right)q^s}, \; p \neq q$$

$$= \frac{n-(n-1)s}{(n+1)-ns}, \; p = q$$

(b) At a one man barber shop, the customer arrive according to Poisson distribution with a mean arrival rate of 5 per hour and his hair cutting time was exponentially distributed with an average hair cut taking 10 minutes. It is assumed that customers were always willing to wait. Calculate the following :

(i) Average number of customers in the shop and the average number of customers waiting for a hair cut.

(ii) The percent of customers who do not have to wait prior to getting into the barber's chair. (5½,4)