2041   *Your Roll No.* ....

B.Sc. (Hons.) / III   **E**

STATISTICS – Paper XXV

(Econometrics)

(Admissions of 1999 and onwards)

Time : 2 Hours   *Maximum Marks : 38*

*(Write your Roll No. on the top immediately
on receipt of this question paper.)*

Answer **Four** *questions in all,
selecting* **two** *questions from each Section.*

SECTION I

1. (a) Consider a linear parametric function $c'\beta$ where c
    is a k x 1 column vector of known constants. Let
    $c'\hat{\beta}$ be an unbiased estimator of $c'\beta$. Prove that
    $c'\hat{\beta}$ is the best linear unbiased estimator of $c'\beta$
    among the class of all linear unbiased estimators
    of $c'\beta$. Also obtain an interval predictor for $c'\beta$.

    (b) Consider the generalized least squares model
    $Y = X\beta + u$ with $E(uu') = \sigma^2 \Omega$, σ^2 being unknown
    and Ω being known (symmetric positive definite
    matrix) is

                                           P.T.O.

$$\Omega = \begin{bmatrix} \dfrac{1}{\lambda_1} & 0 & \cdots & 0 \\[2ex] 0 & \dfrac{1}{\lambda_2} & \cdots & 0 \\[2ex] \vdots & \vdots & \vdots & \vdots \\[2ex] 0 & 0 & \cdots & \dfrac{1}{\lambda_n} \end{bmatrix}$$

where λ's being known positive numbers. Further, suppose that the variance of the disturbance term is proportional to $1/X$ i.e. $E(u_i^2) = \sigma^2 / X_i$. Find out the variance covariance matrices of β_{ols} and $b_{Aitkens}$ for the case of single explanatory variable.

(4½,5)

2. (a) What are distributed lag models ? Discuss adaptive expectation model in details.

   (b) What do you understand by multicollinearity ? For a simple model $Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + U_t$; show that the precision of the estimation falls i.e. (i) these errors may be highly correlated and (ii) sampling variance of the coefficients will be large.   (4,5½)

3. (a) Define auto-correlation. Discuss and show, what happens if ordinary least square estimation is applied to the model suffering from auto-correlated disturbances ?

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(b) Explain the procedure of exponential smoothing. Also show that :

(i) smoothed statistic S_T is a weighted average of all past observations;

(ii) for sufficiently large T the exponential smoothing process yields an unbiased estimator of the true process average.

$$(5\frac{1}{2}, 4)$$

SECTION II

4. (a) What is a Lorenz curve ? For the distribution of income given by Pareto's law, derive the number of persons having income greater than or equal to unity, their average income and hence the curves of concentration. Also define concentration ratio and compute concentration ratio for the above distribution when $v = 1.5$.

(b) The price elasticity of a demand curve $x = f(p)$ is of the form $(1 - 2p)(3 - 2p)$. Find the demand curve. $(6\frac{1}{2}, 3)$

5. (a) The demand functions of two commodities X_1 and X_2 are $p_1 = 16 - x_1^2$ and $p_2 = 8 - x_2$ and joint-cost function is $C = 10 + 4x_1 + 2x_2$. Determine the quantity that maximizes the profit of monopolist and also find the maximum profit.

P.T.O.

(b) Suppose that a consumer has Rs. 50, to be divided between commodities X and Y and suppose that the unit price of Y is fixed at Rs. 0.30. What will be his demand equation for X if his utility function is $U = \log x + 3 \log y$ (where x is the amount consumed of X and y is the amount consumed of Y)? (5½,4)

6. (a) What is Cobb-Douglas production function ? Show that its isoquants are negatively sloped but the rate of change of tangent to the isoquants is positive. Define Marginal rate of substitution and elasticity of substitution. Show that the elasticity of substitution for this function is constant.

(b) When are the two commodities said to be complimentary or competitive ?

Suppose the demand functions of two commodities are given by $x_1 = p_1^{-3} e^{4p_2+7}$ and $x_2 = p_2^{-3} e^{-3p_1+4}$ respectively, where p_1 and p_2 are the prices and x_1 and x_2 are the quantities of the two commodities respectively. Comment on the nature of the commodities. Also find four partial elasticities of demand. (5,4½)