[This question paper contains 2 printed pages.]

Sr. No. of Question Paper : 1150 **G** **Your Roll No...............**

Unique Paper Code : 237501

Name of the Paper : Statistical Inference I (STHT-501)

Name of the Course : **B.Sc. (H) Statistics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all selecting **three** from each section.

SECTION I

1. (a) Explain the concepts of consistency and unbiasedness. Give an example of an estimator

   (i) which is consistent but not unbiased.

   (ii) which is unbiased but not consistent.

   (b) State and prove invariance property of consistent estimators. Hence obtain a consistent estimator of $\theta^2 + \theta - \sqrt{\theta}$, when X follows Poisson distribution with parameter θ. (6,6½)

2. (a) Define MVB and MVU estimators. Obtain MVB estimator of θ in case of a random sample of size n from N(μ, θ), where μ is known.

   (b) Let T_0 be MVU estimator and T_1 be an unbiased estimator with efficiency e. Show that no unbiased linear combination of T_0 and T_1 can be a MVU estimator. (7½,5)

3. (a) State and prove Cramer-Rao inequality and explain its significance.

   (b) Obtain an unbiased estimator for θ^2 in case of a random sample of size n drawn from the binomial distribution with p.m.f. :

$$p(x) = \binom{m}{x} \theta^x (1-\theta)^{m-x}; \quad x = 0, 1, 2, \text{------}, m.$$ (7½,5)

P.T.O.

SECTION – II

4. (a) Let $Y_1 < Y_2 < Y_3 < Y_4 < Y_5$ be the order statistics of a random sample of size 5 from the uniform distribution $U(0,\theta)$. Show that $2Y_3$ is an unbiased statistic for θ. Determine the joint probability density function of Y_3 and the sufficient statistic Y_5 for θ. Find the conditional expectation $E(2Y_3 \mid Y_5)$ and compare its variance with that of $2Y_3$.

   (b) State and prove Factorisation theorem on sufficiency. (6,6½)

5. (a) Define completeness of a statistic. Show that the family of $N(\theta,\sigma^2)$ distributions, where σ^2 is known, is complete. Hence obtain MVU estimator of θ.

   (b) Show that the most general form of the distribution for which the sample arithmetic mean is the m. l. estimator of θ has the p.d.f. :

$$f(x, \theta) = \exp[(x - \theta) A'(\theta) + A(\theta) + B(x)],$$

   where $A(\theta)$ and $B(x)$ are arbitrary functions of θ and x respectively. (6½,6)

6. (a) State optimum properties of maximum likelihood estimators.

   (b) Show that in sampling from the distribution with p.d.f. :

$$f(x, \theta) = \theta e^{-\theta x}, \quad 0 < x < \infty.$$

   $\frac{1}{x}$ is the m.l. estimator of the parameter θ. Also show that $E\left[\frac{n-1}{n\bar{x}}\right] = \theta$ and compare the variance of the m.l. estimator with that of variance of unbiased estimator. (6½,6)

7. (a) State and prove Rao-Blackwell theorem and explain its significance in point estimation.

   (b) Obtain 100 $(1-\alpha)$% confidence interval for difference of binomial proportions when $X_1, X_2, \text{------}, X_m$ is a random sample from $B(m, p_1)$ and $Y_1, Y_2, \text{------}, Y_n$ is a random sample from $B(n, p_2)$, for large m and n. (6½,6)

8. (a) Describe the procedure of obtaining estimators by the method of minimum Chi-square.

   (b) A random variable X takes the values 0, 1, 2, with respective probabilities

$$\frac{\theta}{4N}+\frac{1}{2}\left(1-\frac{\theta}{N}\right),\ \frac{\theta}{2N}+\frac{\alpha}{2}\left(1-\frac{\theta}{N}\right),\ \frac{\theta}{4N}+\frac{1-\alpha}{2}\left(1-\frac{\theta}{N}\right),$$

   where N is a known number and α and θ are unknown parameters. Estimate θ and α by the method of moments based on a random sample of size n. (6½,6)

(600)