[This question paper contains 2 printed pages.]

Sr. No. of Question Paper : 1152 **G** **Your Roll No...............**

Unique Paper Code : 237503

Name of the Paper : STHT-503 : Linear Models

Name of the Course : **B.Sc. (H) Statistics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all, selecting **three** questions from each Section.

SECTION I

1. (a) Consider the multiple linear regression model $\left(\underset{\sim}{Y}, X\underset{\sim}{\beta}, \sigma^2 I\right)$. Obtain an unbiased estimator of σ^2 when X is :

   (i) full rank matrix; and

   (ii) not a full rank matrix

   (b) For the multiple regression model derive the test statistics for testing the significance of individual regression coefficients. (8,4½)

2. Derive the analysis of variance table for oneway classified data under fixed effects model. Also obtain the analysis of covariance table for the same setup. (12½)

3. Suppose $\underset{\sim}{Y} = (Y_1, Y_2, ..., Y_n)'$ to be a vector of n independent standard normal variates then a necessary and sufficient condition for $\underset{\sim}{Y}' A \underset{\sim}{Y}$ to be distributed as chi-square variate with k.d.f. is that A is an idempotent matrix of rank k. Also compute the variance of $\underset{\sim}{Y}' A \underset{\sim}{Y}$. (12½)

4. (a) For a simple linear regression model, develop a test for lack of fit.

(b) Let $\underset{\sim}{Y} \sim N_3(\underset{\sim}{O}, I)$ and let $A = \frac{1}{3}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$

(i) Find expected value of $\underset{\sim}{Y}' A \underset{\sim}{Y}$

(ii) Are $\underset{\sim}{Y}' A \underset{\sim}{Y}$ and $y_1 + y_2 + y_3$ independent ?

(iii) Are $\underset{\sim}{Y}' A \underset{\sim}{Y}$ and $2y_1 + y_2$ independent ? (5,2½,2½,2½)

SECTION II

5. (a) What do you mean by bias in regression estimates ? Suppose we postulate the model $E(y) = \beta_0 + \beta_1 X$ but the true model is $E(y) = \beta_0 + \beta_1 X + \beta_{11} X^2$. Calculate the biases in the least squares estimators of β_0 & β_1 by taking observations at $X = -1, 0, 1$.

(b) What is a parametric function ? Derive a necessary and sufficient condition for which parametric function is estimable. (6½,6)

6. (a) Develop a prediction interval for the future observation y_0 corresponding to a specified level x_0 of the regressor variable x in the simple linear regression model.

(b) Consider the model $E(y_1) = \beta_1 + \beta_2$, $E(y_2) = \beta_1 - \beta_2$ and $E(y_3) = \beta_1 + 2\beta_2$ with usual assumptions. Obtain the BLUE of $\beta_1 - 2\beta_2$ and its variance. (6½,6)

7. Consider the simple linear regression model $Y = \beta_0 + \beta_1 x + \epsilon$ with $E(\epsilon) = 0$, $V(\epsilon) = \sigma^2$ and show that

(i) $\text{Cov}\left(\hat{\beta}_0 \hat{\beta}_1\right) = \frac{-\bar{X}\sigma^2}{S_{xx}}$

(ii) $\text{Cov}\left(\bar{Y}, \hat{\beta}_1\right) = 0$

(iii) $E(MSR) = \sigma^2 + \beta_1^2 S_{xx}$

(iv) $E(MSE) = \sigma^2$ (2½,2,4,4)

8. Write short notes on any **two** of the following :

(i) Partial F-test and sequential F-test.

(ii) R^2 and Adjusted R^2 Statistic.

(iii) No-intercept model. (6,6½)