[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1153 **G** **Your Roll No................**

Unique Paper Code : 237504

Name of the Paper : Stochastic Processes (STHT-504)

Name of the Course : **B.Sc. (Hons.)**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **TWO** questions from each section.
3. Each question carries equal marks.
4. Use of simple calculator is allowed.
5. The questions paper contains **THREE** sections, each carrying 25 marks.

SECTION I

1. (a) Let X have a zero truncated Poisson distribution with

$$P(X=k)=\frac{\left(e^{a}-1\right)^{-1}a^{k}}{k!},\quad k=1,2,......$$

Determine the following :

(i) Probability generating function P(s) of X,

(ii) E(X),

(iii) Var(X),

(iv) Verify that P(1) = 1.

(b) Let X be a non - negative integral valued random variable with $p_n = P(X = n)$; $n = 0,1,2,......$ Obtain the generating functions of :

(i) $P\{X < n\}$,

(ii) $P\{X \geq n\}$. (6,6½)

2. (a) Consider the process $\{X(t), t \in T\}$ whose probability distribution under a certain condition is given by :

$$P(X(t) = n) = \frac{(at)^{n-1}}{(1+at)^{n+1}}, \quad n = 1,2,....$$

$$= \frac{at}{1+at}, \quad n = 0.$$

Is the process $\{X(t), t \in T\}$ stationary or evolutionary ?

(b) Let $S_N = X_1 + X_2 + ... X_N$, where X_i's are i.i.d. random variables and N is a random variable independent of X_i's. Show that

$Var(S_N) = E(N)\ Var(X) + Var(N)\ [E(X)]^2$ and

$$\frac{Var(S_N)}{E(S_N)} = \frac{Var(X)}{E(X)} + \frac{E(X)Var(N)}{E(N)}.$$ (6,6½)

3. (a) Define G.W. branching process assuming that the process starts with a single ancestor. Let the distribution of the number of offsprings be geometric with :

$p_k = P(X = k) = b(1-b)^k$, $k = 0, 1, 2,......(0< b < 1)$. Obtain the p.g.f. of the offspring distribution and hence obtain the probability of ultimate extinction.

(b) Let X be a random variable denoting the number of tosses required to get two consecutive heads when a fair coin is tossed. Show that the pg.f. of

X is $\frac{s^2}{4}\left\{1-\frac{s}{2}-\left(\frac{s}{2}\right)^2\right\}^{-1}$. (6,6½)

SECTION II

4. (a) Suppose that a fair die is tossed. Let the states of X_n be k (k=1,2,...,6), where k is the maximum number shown in the first n tosses. Find the transition probability matrix P and calculate $P(X_2 = 6)$.

(b) Prove that in an irreducible Markov chain, all the states are of the same type. (6,6½)

5. (a) Let the Markov chain consisting of the states 0, 1, 2, 3 have the following transition matrix

$$P = \begin{bmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

Find the closed sets, if any and hence classify the states of the above Markov chain.

(b) Define a classical ruin problem and obtain the following probabilities :

(i) The probability of gambler's ultimate ruin;

(ii) The probability of his winning. (6,6½)

6. (a) Define transient, persistent and periodic recurrent events. Find a necessary and sufficient condition for an event E to be persistent.

(b) Consider a Markov chain with state space {0, 1}, having transition probability matrix

$$P = \begin{bmatrix} 1-(1-c)p & (1-c)p \\ (1-c)(1-p) & (1-c)p+c \end{bmatrix},$$

$0 < p < 1, 0 \le c \le 1$, and initial distribution $p_1 = P(X_0 = 1) = p = 1 - P(X_0 = 0)$. Find (i) $p_n = P(X_n = 1)$, (ii) $E(X_n)$, (iii) $Var(X_n)$, (iv) $Cov(X_{n-1}, X_n)$, (v) $Cov(X_{n-2}, X_n)$ and (vi) $Corr(X_{n-2}, X_n)$. (6,6½)

SECTION III

7. (a) Prove that the interval between two successive occurrences of a Poisson process $\{N(t), t \geq 0\}$ having parameter λ has a negative exponential distribution with mean $\frac{1}{\lambda}$.

(b) Write the differential-difference equation for the Yule-Furry process, starting with one individual. Obtain the p.g.f. and identify the distribution. Hence find the mean and variance. (6,6½)

8. (a) A super market has a single cashier. During the peak hours, customers arrive at a rate of 20 customers per hour. The average number of customers that can be processed by the cashier is 24 hours. Determine the following :

(i) The probability that the cashier is idle.

(ii) The average number of customers in the queuing system.

(iii) The average time the customer spends in the system.

(b) For the linear growth process, starting with i individuals at time 0, obtain (i) the mean population size (ii) the probability of ultimate extinction, when birth rate is less than death rate. (6,6½)

9. (a) If $\{N(t)\}$ is a Poisson process then obtain the auto correlation coefficient between $N(t)$ and $N(t+s)$.

(b) Consider (M/M/1) queuing model with queue discipline FIFO and infinite system capacity. Obtain the steady state probability that there are n customer in the system. Also find (i) the average number of customers in the queue, (ii) variance of the number of customers in the system. (6,6½)