Your Roll No. ....

Sl. No. of Ques. Paper	: **1388**	**F-7**
Unique Paper Code	: **2371501**	
Name of Paper	: **Statistical Inference – I**	
Name of Course	: **B.Sc. (Hons.) Statistics (Erstwhile FYUP)**	
Semester	: **V**	
Duration	: **3 hours**	
Maximum Marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any five questions.

1. (a) Let $X_1, X_2, ..., X_n$ be a random sample from $N(\mu, \sigma^2)$. Examine whether:

$$T=\frac{1}{n}\sum_i |X_i-\mu|$$

is unbiased for σ. If not, obtain un unbiased estimator of σ. Also find efficiency of this unbiased estimator.

(b) State and prove Cramer-Rao inequality. Under what conditions does equality hold? Explain its significance. 7,8

2. (a) Let $X_1, X_2, ..., X_n$ be a random sample from exponential distribution with p.d.f.:

$$f(x,\theta)=\frac{1}{\theta}e^{-x/\theta}, \quad x>0, \theta>0$$

Obtain MVB estimator for θ. Hence find the variance of MVB estimator.

(b) State and prove sufficient conditions for consistency. In a random sample of size n from $N(\mu, \sigma^2)$, obtain consistent estimator of σ^2 when μ is known. 6,9

3. (a) State and prove Factorization theorem for the existence of sufficient statistic.

What is the advantage of this criterion over Fisher-Neyman criterion?

(b) Let $Y_1<Y_2< ....<Y_n$ be the order statistics of a random sample of size n from the uniform distribution having p.d.f. :

$$f(x,\theta)=\begin{cases}\frac{1}{\theta}, & 0\le x\le\theta \\ 0, & \text{otherwise}\end{cases}$$

Show that Y_n is complete sufficient for θ. Hence obtain MVU estimator for θ. 7,8

4. (a) Show that MVU estimator is unique. Let T_1 and T_2 be two unbiased estimators for θ with variances σ_1^2 and σ_2^2 (both known) and correlation coefficient ρ_θ. For what value of α does

$$T=\alpha T_1+(1-\alpha)T_2$$

have minimum variance? Find the variance of T.

(b) Explain the method of minimum Chi-square and modified minimum Chi-square. Under what conditions is it identical with the method of maximum likelihood estimation? 8,7

5. (a) Describe method of moments and find estimator of θ by the method of moments for:

$$f(x,\theta)=\begin{cases}\frac{1}{2}e^{-|x-\theta|}, & -\infty<x<\infty\\ 0, & \text{otherwise}\end{cases}$$

(b) Explain the procedure of estimating the parameters by the method of maximum likelihood. Also mention all the properties of ML estimators. 7,8

6. (a) If X_1, X_2 is a random sample of size 2 from a distribution having p.d.f. :

$$f(x,\theta)=\frac{1}{\theta}e^{-x/\theta},\ 0<x<\infty,$$

show that $Y_1=X_1+X_2$ is sufficient estimator for θ. Further show that $Y_2=X_2$ is an unbiased estimator for θ with variance θ^2. Find $E(Y_2|Y_1=y_1)$ and compare its variance with that of Y_2.

(b) In sampling from a Power Series distribution with probability function:

$$f(x,\theta)=\frac{a_x\theta^x}{\phi(\theta)},\ x=0,1,2\ ....$$

where a_x may be zero for some x. Show that ML estimator of θ is the root of the equation:

$$\bar{x}=\frac{\theta\phi'(\theta)}{\phi(\theta)}=\mu(\theta)\ \text{or}\ \mu(\theta)=\bar{x}$$

8,7

7. (a) Distinguish between point estimation and interval estimation.Let $X_1, X_2, ..., X_n$ be a random sample of size n from rectangular distribution with pd.f.

$$f(x,\theta)=\begin{cases}\frac{1}{\theta}, & 0\le x\le\theta\\ 0, & \text{otherwise}\end{cases}$$

If R be the sample range and ε is given by $\varepsilon^{n-1}[n-(n-1)\varepsilon]=\alpha$, show that R and $\frac{R}{\varepsilon}$ are confidence limits for θ with confidence coefficient $(1-\alpha)$.

(b) Explain the method of constructing the confidence intervals for large samples by using likelihood approach. Using this approach, obtain $100(1-\alpha)\%$ confidence limits for the parameter θ of Poisson distribution. 7,8