[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1211 **E** **Your Roll No................**

Unique Paper Code : 237601

Name of the Course : **B.Sc. (H) Statistics**

Name of the Paper : Statistical Inference II (STH–601)

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all selecting **three** from each section.
3. Attempt all parts of a question in continuation.

SECTION I

1. (a) Define the following terms :

(i) Sample space and parametric space with examples.

(ii) Critical region.

(iii) Size and power of the test.

(b) In a shipment of 10 articles, θ are defective. The hypothesis $H_0: \theta = 5$ is rejected in favour of $H_1: \theta = 6$ if two articles selected at random with replacement are of the same type either both defective or both non defective. Determine the size of the critical region and power of the test.

P.T.O.

(c) State and prove Neyman-Pearson lemma for the construction of MP critical region. (4½,3,5)

2. (a) Let $X_1, X_2, ----, X_n$ be independent random variables such that each X_i is distributed as $N(\alpha_i\theta, \alpha_i\sigma^2)$, $\forall$ i = 1,2, --, n, where σ^2 and α_i are known positive quantities. Obtain the B.C.R. of size α for testing H_0: $\theta = \theta_0$ against $H_1 : \theta = \theta_1$ $(\neq\theta_0)$.

(b) Define UMPU critical region. Prove that if W is an MP region for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$, then it is necessarily unbiased. Also prove that the same holds good if W is an UMP region. (6,6½)

3. (a) Let $X_1, X_2, ----, X_n$ be a random sample from p.d.f.

$$f(x, \theta) = \theta x^{\theta-1}, \ 0 \le x \le 1, \ \theta > 0.$$

Find UMP tests of size α in terms of chi-square statistics for testing H_0: $\theta = \theta_0$ against one-sided alternatives.

(b) Show that the likelihood ratio test for testing $H_0 : \alpha = 0$ against $H_1 : \alpha \neq 0$ based on a random sample of size n from a population with p.d.f.

$$f(x; \alpha; \beta) = \frac{1}{2\beta}; \quad \alpha - \beta \le x \le \alpha + \beta$$

is $(R/2Z)^n$, where $R = X_{(n)} - X_{(1)}$ and $Z = \max[-X_{(1)}, X_{(n)}]$. (8½,4)

4. Construct likelihood ratio test for testing $H_0 : \theta = \theta_0$ against various alternatives in case of a random sample of size n drawn from a $N(\theta, \sigma^2)$, where σ^2 is unknown. (12½)

SECTION II

5. (a) Explain the difference between Neyman – Pearson test procedure and sequential probability ratio test procedure (SPRT) of hypothesis testing.

For the SPRT of strength

(α_1, β_1) and for given $A = \frac{1-\beta}{\alpha}$ and $B = \frac{\beta}{1-\alpha}$ prove that

(i) $\alpha_1 \leq \frac{\alpha}{1-\beta}$, $\beta_1 \leq \frac{\beta}{1-\alpha}$

(ii) $\alpha_1 + \beta_1 \leq \alpha + \beta$

(b) Explain the Kruskal-Wallis test for testing the null hypothesis that K Independent samples are from the identical populations. (7½,5)

6. (a) Let X follows $N(\mu, \sigma^2)$, μ being known. Develop SPRT for testing H_0 : $\sigma^2 = \sigma_0^2$ against H_1: $\sigma^2 = \sigma_1^2$ $(\sigma_1^2 > \sigma_0^2)$. Also obtain its OC and ASN functions.

(b) Discuss the Kolmogorov-Smirnov one sample test for goodness of fit. (7,5½)

7. Develop the runs test for testing whether two given samples are drawn from the same continuous population. Discuss the case of ties. How can this test be utilized to test the randomness of a given set of observations ? (12½)

8. Write short notes on any **three** of the following :

(i) Likelihood ratio test and its properties.

(ii) SPRT procedure and its O.C. and A.S.N. functions.

(iii) Non parametric tests and their advantages over parametric tests.

(iv) Median test. (4,4½,4)