[This question paper contains 4 printed pages.]

4413 *Your Roll No.* ....

Subsidiary for B.Sc. Honours/I **AS**

MATHEMATICS – Paper I

(Differential Calculus and Integral Calculus)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **Six** *questions in all, selecting at least* **two** *questions from each Section.* **All** *questions carry equal marks.*

SECTION I

1. (a) If $u = \tan^{-1} \dfrac{x^3 + y^3}{x - y}$, then show that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \left(1 - 4 \sin^2 u\right) \sin 2u$$

(6)

(b) Show that the normal at any point of the curve

$$x = a\cos\theta + a\theta\sin\theta, \quad y = a\sin\theta - a\theta\cos\theta$$

is at a constant distance from the origin. (6½)

2. (a) If $y = (\sin^{-1}x)^2$, then prove that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$$

Also find $y_n(0)$. (6)

P.T.O.

(b) Find the values of x for which $\sin x - x\cos x$ is a maximum or a minimum. (6½)

3. (a) Verify Rolle's Theorem for the function

$$f(x) = (x-a)^m (x-b)^n, \ x \in [a, b]$$

where m and n are positive integers. (6)

(b) Evaluate

$$\lim_{x \to 0}\left(\frac{1}{x} - \frac{1}{e^x - 1}\right)$$ (6½)

4. (a) Find the asymptotes of the curve

$$y^3 + x^2y + 2xy^2 - y + 1 = 0$$ (6)

(b) Find the radius of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ and prove that it is minimum at the point $\left(\frac{a}{4}, \frac{a}{4}\right)$. (6½)

5. (a) Find the position and nature of the double points of the curve

$$x^4 - 4ax^3 + 2ay^3 + 4a^2x^2 - 3a^2y^2 - a^4 = 0.$$ (6)

(b) Trace the curve

$$y^3 = a^2x - x^3$$ (6½)

SECTION II

6. Evaluate the following integrals :

(i) $\int \frac{x}{\sqrt{8+x-x^2}}$ (6½)

(ii) $\int \frac{1+\sin 2x}{1+\cos 2x} e^{2x}$ (6)

7. (a) Show that the area of the loop of the curve $y^2(a+x) = x^2(3a-x)$ is equal to the area between the curve and its asymptote. (6)

(b) Find the length of the arc of the parabola $y^2 = 4ax$ cut off by its latus rectum. (6½)

8. (a) If $I_n = \int_0^{\pi/4} \tan^n \theta \, d\theta$, then show that

$$n(I_{n-1} + I_{n+1}) = 1.$$ (6)

(b) Show that

$$\int_0^1 x(1-x)^n dx = \frac{1}{(n+1)(n+2)}.$$ (6½)

9. (a) Prove that the volume of the solid generated by the revoltion of the curve $y = \frac{a^3}{a^2+x^2}$ about its asymptot is $\frac{\pi^2 a^3}{2}$. (6)

(b) Find the surface of the solid formed by revolving the cardioide $r = a(1 + \cos\theta)$ about the initial line. (6½)

10. (a) Evaluate $\iint xy\,(x^2 + y^2)^{n/2}\,dx\,dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$. (6)

(b) Evaluate

$$\iiint \frac{dx\,dy\,dz}{x^2 + y^2 + (z-2)^2}$$

over the sphere $x^2 + y^2 + z^2 \le 1$. (6½)