[This question paper contains 4 printed pages.]

4415 *Your Roll No.* ....

Subsidiary for B.Sc. Honours/II **AS**

MATHEMATICS – Paper III

(Analytic Geometry of Two Dimensions and Vectors)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any **six** *questions.*
All *questions carry equal marks.*

1. (a) Find the equation of the pair of straight lines obtained by joining the origin to the points of intersection of the straight line $y = mx + c$ and the circle $x^2 + y^2 = a^2$, and prove that these are at right angles if $2c^2 = a^2(1 + m^2)$.

   (b) Show that the product of the perpendiculars from the point (x', y') on the lines

   $$ax^2 + 2hxy + by^2 = 0$$

   is equal to

   $$\frac{ax'^2 + 2hx'y' + by'^2}{\sqrt{(a-b)^2 + 4h^2}}$$

2. (a) Find the coordinates of the limiting points of the coaxal system to which the circles

   $x^2 + y^2 + 4x + 2y + 5 = 0$ and $x^2 + y^2 + 2x + 4y + 7 = 0$ belong.

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(b) Show that the circle on the chord

$$x\cos\alpha + y\sin\alpha - p = 0$$

of the circle $x^2 + y = a^2$ as diameter is

$$x^2 + y^2 - a^2 - 2p\ (x\cos\alpha + y\sin\alpha - p) = 0$$

3. (a) Find the locus of the point of intersection of two normals to the parabola $y^2 = 4ax$ which are at right angles to one another.

(b) Prove that the orthocentre of any triangle formed by three tangents to a parabola lies on the directrix.

4. (a) Show that locus of the middle point of normal chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is given by the equation

$$\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^2 \left(\frac{a^6}{x^2} + \frac{b^6}{y^2}\right) = \left(a^2 - b^2\right)^2$$

(b) CP, CQ are conjugate semi-diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, and the circles with CP and CQ as diameters intersect at R, show that R lies on the curve

$$2(x^2 + y^2)^2 = a^2x^2 + b^2y^2$$

5. (a) If a circle and the rectangular hyperbola $xy = c^2$ meet in four points t_1, t_2, t_3 and t_4, prove that $t_1 t_2 t_3 t_4$ = constant.

(b) Find the equation of the hyperbola and its conjugate, which has

$$3x - 4y + 7 = 0 \text{ and } 4x + 3y + 1 = 0$$

for asymptotes and pass through the origin.

6. Trace the conic

$$9x^2 + 24xy + 16y^2 - 2x + 14y + 1 = 0.$$

7. (a) PSP′ and QSQ′ are two perpendicular focal chords of a conic; prove that

$$\frac{1}{SP.SP'} + \frac{1}{SQ.SQ'} \text{ is constant.}$$

(b) PSP′ is a focal chord of a conic. Prove that the angle between the tangents at P and P′, is

$$\tan^{-1}\left(\frac{2e\sin\alpha}{1-e^2}\right)$$

where α is the angle between chord and the initial line.

8. (a) Find the area of a triangle whose two adjacent sides are given by

$$\vec{a} = \hat{i} + 4\hat{j} - \hat{k}, \quad \vec{b} = \hat{i} + \hat{j} + 2\hat{k}$$

(b) Prove that

$$\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = \vec{0}$$

9. (a) Evaluate

$$\frac{d}{dt}\left(\vec{v}\cdot\frac{d\vec{v}}{dt}\times\frac{d^2\vec{v}}{dt^2}\right)$$

where $\vec{v}$ is a differentiable function of a scalar t.

(b) A particle moves so that its position vector is given by $\vec{r} = \cos wt\,\hat{i} + \sin wt\,\hat{j}$ where w is a constant. Show that the velocity $\vec{v}$ of the particle is perpendicular to $\vec{r}$ and $\vec{r}\times\vec{v}$ is a constant vector.

10. (a) If $\vec{f}(t) = \hat{i} + (t^2 - 2t)\hat{j} + (3t^2 + 3t^3)\hat{k}$, then evaluate.

$$\int_0^1 \vec{f}(t)\,dt.$$

(b) Find the total work done in moving a particle in a force field given by

$\vec{F} = 3xy\,\hat{i} - 5z\,\hat{j} + 10x\,\hat{k}$ along the

curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$.