[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 617 **E** **Your Roll No...............**

Unique Paper Code : 235485

Name of the Course : **B.A. (H)**

Name of the Paper : Mathematics – III (Elements of Analysis)

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. There are **3** sections. Attempt **all** the sections.

SECTION I

(Attempt any **three** *questions.)*

1. (a) Define supremum and infimum of a set of real numbers.

   Find the supremum and infimum of

   (i) $\left\{\frac{1}{n} : n \in N\right\}$

   (ii) $\left\{(-1)^n : n \in N\right\}$ (1,1,1½,1½)

   (b) If $(a_n) \to 0$ and (b_n) is bounded, then prove that $(a_n b_n) \to 0$. (5)

2. (a) Show that the sequence (2^n) does not converge. (5)

   (b) If (a_n) and (b_n) are two convergent sequences with $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$, show that the sequence $(a_n + b_n)$ converges to $(a + b)$. (5)

3. (a) State Cauchy's first theorem on limits.

Hence, show that the sequence (a_n), defined as

$a_n = \frac{1}{n}\left(1+\frac{1}{2}+ -- +\frac{1}{n}\right)$, is convergent. (1,4)

(b) Show that the sequence (a_n), defined as

$$a_n = \frac{1}{1!}+\frac{1}{2!}+----+\frac{1}{n!}$$

is a convergent sequence. (5)

4. (a) State Squeeze Theorem.

Use this to show that $\left(\frac{\sin n}{n}\right)$ converges to zero. (1,4)

(b) Let $a_1 = 1$, $a_{n+1} = \sqrt{2+a_n}$ $\forall$ $n \geq 1$. Show that the sequence (a_n) is convergent. Also find its limit. (4,1)

SECTION II

(Attempt any **two** *questions.)*

5. (a) Let $\sum_{n=1}^{\infty} u_n$ and $\sum_{n=1}^{\infty} v_n$ be two series of positive terms such that $u_n < Kv_n$ for all $n \in N$, where K is a fixed positive number. Then prove that if $\sum_{n=1}^{\infty} v_n$ converges, so does $\sum_{n=1}^{\infty} u_n$ and if $\sum_{n=1}^{\infty} u_n$ diverges then $\sum_{n=1}^{\infty} v_n$ also diverges. (6½)

(b) Test the convergence of the series :

$\frac{1^2.2^2}{1!}+\frac{2^2.3^2}{2!}+\frac{3^2.4^2}{3!}+\ldots.$ (6)

6. (a) State Cauchy's general principle of convergence for an infinite series of real numbers. Use it to show that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ is not convergent. (2,4½)

(b) Test for the convergence of the following series:

$$\sqrt{\left(\frac{1}{4}\right)} + \sqrt{\left(\frac{2}{6}\right)} + \sqrt{\left(\frac{3}{8}\right)} + \ldots + \sqrt{\left(\frac{n}{2(n+1)}\right)} + \ldots \quad (6)$$

7. (a) Test for the convergence of the series whose n^{th} term is $\left(1+\frac{1}{\sqrt{n}}\right)^{\left(-n^{3/2}\right)}$. (6½)

(b) Test for the convergence and absolute convergence of the series :

$$\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{7}} + \ldots \quad (6)$$

SECTION III

(Attempt any **two** *questions.)*

8. Determine the radius of convergence and interval of convergence for the following power series :

(a) $\sum_{n=1}^{\infty} \frac{3^{-n}}{n} x^{2n}$

(b) $\sum_{n=1}^{\infty} \frac{1}{n}(x-1)^{n}$ (5,5)

9. (a) State and prove Cauchy Hadamard Theorem. (1,5)

(b) Discuss the convergence of the power series $\sum_{n=1}^{\infty} \frac{1}{n^2} x^n$ in $[-1, 1]$. (4)

10. (a) Using the power series representation

$$\frac{1}{(1-x)^2} = 1+2x+3x^2+\ldots = \sum_{n=1}^{\infty} nx^{n-1},\ |x|<1$$

Evaluate $\sum_{n=1}^{\infty} \frac{n}{k^n}$, for a fixed integer $k > 1$. (5)

(b) Suppose that, the power series $\sum_{n=0}^{\infty} \alpha_n x^n$, has radius of convergence 2. Find radius of convergence of the power series $\sum_{n=0}^{\infty} \alpha_n x^{kn}$, where k is a fixed positive integer. (5)