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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **36**

Unique Paper Code : **237262** **E**

Name of the Paper : **STP-202 : Statistical Methods–I**

Name of the Course : **B.Sc. (Mathematical Sciences)**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *six* questions

All questions carry equal marks.

1. (*a*) The p.d.f. of a continuous random variable X is given by :

$$f(x) = k(x+1), -1 < x < 1$$

(*i*) Find k

(*ii*) Determine the number 'b' such that :

$$P(X < b) = P(X \geq b).$$

(*b*) If X is a random variable with moment generating function :

$$M_x(t), \text{ and } \mu_r' = E(X^r)$$

exists, then show that :

$$\mu_r' = \left|\frac{\partial^r M_x(t)}{\partial t^r}\right|_{t=0} ; r = 1, 2, \ldots$$ 6,6½

P.T.O.

2. (*a*) State and prove addition theorem of expectation for two random variables. Hence find mean of $Y = 2X_1 - 3X_2$, where X_1 and X_2 are random variables with means 3 and 5 respectively.

(*b*) The joint probability density function of a two dimensional random variable (X, Y) is given by :

$$f(x) = \begin{cases} kx^2 y, & 0 < y < 1, 0 < x < 1 \\ 0 & , \text{elsewhere} \end{cases}$$

(*i*) Determine the constant k

(*ii*) Find the marginal density function of X.

(*iii*) Find :

$P(X \leq 1/2)$, $P(0 < X < 3/4, 1/3 < Y < 2)$. 4½, 8

3. (*a*) Show that for uniform distribution :

$$f(x) = \frac{1}{2a}, \; -a < x < a$$

m.g.f. about origin is $\frac{1}{at}\sinh(at)$. Also show that moments of even order are given by :

$$\mu_{2n} = \frac{a^{2n}}{(2n+1)} \; ; \; n = 0, 1, 2, \ldots$$

(*b*) If X has a binomial distribution with parameter n and p, then prove that :

$P(X \text{ is even}) = \frac{1}{2}\,[1 + (q - p)^n]$, where $q = 1 - p$ 7,5½

4. (*a*) Define negative binomial distribution. Show how the moments of negative binomial variate can be written from the corresponding formulae for the binomial variate.

(*b*) Define Beta distribution and obtain its mean and harmonic mean. 6,6½

5. (*a*) Let X be a random variable with p.m.f. :

$$p(x) = pq^x;\ x = 0,\ 1,\ 2, \ldots\ldots;$$

where p and q are positive, $p + q = 1$. Find the m.g.f. of X. Hence show that :

$$m_2 = m_1\ (2m_1 + 1),\ m_1 \text{ and } m_2$$

being the first two moments.

(*b*) Define normal distribution. Obtain its moment generating function and cumulant generating function and hence find its mean and variance. 6,6½

6. (*a*) If X has uniform distribution in [0, 1] find the p.d.f. of $-2\log_e X$. Also identify the distribution.

(*b*) Let X be a random variable with cumulants given by :

$$k_r = n\{(r-1)!\},\ n > 0;\ r = 1, 2, 3, \ldots.$$

Find cumulant generating function of X. Hence, find m.g.f. and characteristic function of X. 6,6½

7. (*a*) The random variable X has an exponential distribution :

$$f(x) = e^{-x}, 0 < x < \infty.$$

Find the density function of the random variable :

(*i*) $Y = 3X + 5$,

(*ii*) $Y = X^3$.

(*b*) Define Gamma distribution. Also state and prove additive property of Gamma distribution. 6,6½

8. (*a*) Let $X_1, X_2, X_3, ...$ be i.i.d Poisson variate with parameter λ. Use Central Limit Theorem (CLT) to estimate $P(120 \leq S_n \leq 160)$, where :

$$S = X_1 + X_2 + X_3 + ... + X_n;\ \lambda = 2 \text{ and } n = 75.$$

(*b*) State Chebyshev's inequality. A symmetric die is thrown 600 times. Find the lower bound for the probability of getting 80 to 120 sixes. 6½,6