This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **621**

Unique Paper Code : **235683**

E

Name of the Paper : **Mathematics-I (Algebra & Calculus)**

Other than Economics

Name of the Course : **B.A. (Hons.)**

Semester : **VI**

Duration : **3 Hours**

Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. **1** is compulsory and carries **15** marks.

Attempt *six* more questions selecting at least *two* questions from each Section. Each question carries **10** marks.

1. (*i*) Find matrix A such that :

$$A\begin{bmatrix} 3 & 4 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 8 \\ 9 & 4 \end{bmatrix}.$$

(*ii*) Evaluate :

$$\lim_{x \to \infty} \frac{2x^3 + 3x^2 + 5}{-5x^3 + 8x - 17}.$$

(*iii*) Find :

$$\frac{dy}{dx},$$

when $y = x^x$.

(*iv*) Show that :

$$\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = (x - y)(y - z)(z - x)(xy + yz + zx).$$

(*v*) Evaluate :

$$\int \cot x \log \sin x \, dx.$$

Section I

2. (*i*) Let

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}.$$

Verify that $A^T . A$ is symmetric.

(*ii*) Solve the following system of linear equations using Cramer's rule :

$$6x + y - 3z = 5$$

$$x + 3y - 2z = 5$$

$$2x + y + 4z = 8.$$

3. (*i*) Find the equation of the ellipse with focus (–1, 1), directrix $x - y + 3 = 0$ and eccentricity is 1/2.

(*ii*) Find the equation of the straight line passing through the point of intersection of the lines $x - y = 1$ and $2x - 3y + 1 = 0$ and parallel to the line $3x + 4y = 14$.

4. (*i*) Find centre and radius of the following circle :

$$x^2 + y^2 + 4x + 2y + 5 = 0.$$

(*ii*) Find the equation of the parabola, whose focus is (2, 2) and directrix is the line $x + 2y - 1 = 0$.

Section II

5. (*i*) If :

$$y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \ldots\ldots + \infty}}},$$

prove that :

$$(2y-1)\frac{dy}{dx} = \frac{1}{x}.$$

(*ii*) Determine the constants a and b so that the function $f(x)$ defined below is continuous everywhere :

$$f(x) = \begin{cases} 2x+1, & \text{if } x \le 1 \\ ax^2+b, & \text{if } 1 < x < 3. \\ 5x+2a, & \text{if } x \ge 3 \end{cases}$$

6. (*i*) Find the maximum value of the function :

$$f(x) = x^{\frac{1}{x}}.$$

(*ii*) Determine the intervals on which the function $f(x) = x^3 - 6x^2 + 9x$ is increasing or decreasing.

7. (*i*) Verify whether the function $f(x) = \sin x$ in $[0, \pi]$ satisfies the conditions of Rolle's theorem and hence find c as prescribed by the theorem.

(*ii*) Write down the Maclaurin series expansion for the function $f(x) = \log(1+x)$.

Section III

8. (*i*) Find the entire length of the astroid :

$$x^{2/3} + y^{2/3} = a^{2/3}.$$

(*ii*) Find :

$$\int \frac{dx}{x[(\log x)^2 - 5\log x + 6]}.$$

9. (*i*) Solve the following differential equation $(1 + x^2)dy = xydx$.

(*ii*) The marginal cost function is $MC = 4 + 2x + x^2$, find the total and average cost functions, given that the fixed cost is Rs. 500.

10. (*i*) Evaluate :

$$\int_0^{\pi/4} \log(1 + \tan x)dx.$$

(*ii*) Test for convergence series :

$$\sum \sin\frac{1}{x}.$$