[This question paper contains 20 printed pages.]

Sr. No. of Question Paper : 1322 **E** **Your Roll No................**

Unique Paper Code : 227202

Name of the Course : **B.A. (H) Economics**

Name of the Paper : Statistical Methods for Economic – II

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Paper is divided into **four** Sections. Attempt **all** Sections. Choice is applicable in Section III and IV only.
3. Answer to **all** question within each Section are to follow each other. Start the answer to each question on a new page and all sub-parts of a question should follow one after the other.
4. Use of simple calculator is permitted.
5. Required statistical tables are attached with this question paper.
6. Answer may be written either in English or Hindi; but the same medium should be used throughout the paper.

छात्रों के लिए निर्देश

1. इस प्रश्न-पत्र के मिलते ही ऊपर दिए गए निर्धारित स्थान पर अपना अनुक्रमांक लिखिए।
2. यह प्रश्न पत्र चार भाग में है। सभी भागों का उत्तर दें। भाग III और IV में चयन उपलब्ध है।
3. प्रत्येक भाग के **सभी** प्रश्नों के उत्तर एक साथ लिखें। हर प्रश्न का उत्तर नए पृष्ठ से शुरू करें और पूरा प्रश्न का उत्तर एक साथ लिखें।
4. साधारण कैलकुलेटर का इस्तेमाल कर सकते हैं।
5. आवश्यक सांख्यिकी तालिकाएं पेपर के साथ संलग्न है।
6. इस प्रश्न-पत्र का उत्तर अंग्रेजी या हिंदी किसी एक भाषा में दीजिए, लेकिन सभी उत्तरों का माध्यम एक ही होना चाहिए।

SECTION – I

(भाग – I)

Attempt **all** *questions..*

सभी प्रश्नों का उत्तर दीजिए ।

1. The weight of a certain kind of screws is a random variable with mean 2/3 mg and variance 1/2 mg. If 100 screws are selected randomly,

   (i) Find the probability that their :

      (a) Total weight is greater than 59.67 mg.

      (b) Average weight is greater than 0.69 mg.

   (ii) Can the probability in part (b) above be calculated if instead a sample of 10 screws was selected ? Why or why not ? (5)

एक निश्चित प्रकार के पेचों का भार एक बेतरतीब चर है, जिसका माध्य 2/3 mg व विचरण 1/2 mg है । यदि 100 पेचों को बेतरतीब ढंग से चुना जाए तो :-

   (i) सम्भावना ज्ञात करें कि

      (अ) इनका कुल भार 59.67 mg से अधिक होगा ?

      (ब) इनका औसत भार 0.69 mg से अधिक होगा ?

   (ii) क्या भाग (ब) में सम्भावना ज्ञात की जा सकती है, यदि 10 पेचों के नमूने का चुनाव किया जाए ? क्यों व क्यों नहीं ?

2. The probability distribution of the number of boxes of chocolates sold (X) on a given day by a seller is given as follows :

X :	0	1
P(X) :	0.2	0.8

Let (X_1, X_2) be a random sample of the number of boxes sold on any two randomly chosen days. Write the sampling distribution of s^2 (sample variance). What is the relationship between $E(s^2)$ and σ^2 ? (5)

एक विक्रेता द्वारा किसी एक दिन पर बिक्री किए गए चॉकलेटों के बॉक्स (X) की संख्या का सम्भावना वितरण निम्नलिखित दिया गया है :-

X :	0	1
P(X) :	0.2	0.8

मान लीजिए कि (X_1, X_2) एक बेतरतीब नमूना है । किन्हीं दो दिनों पर बॉक्सों के बिक्री की गई संख्या का s^2 (नमूना विचरण) का नमूना वितरण लिखिए । $E(s^2)$ व σ^2 के बीच सम्बन्ध क्या है ?

SECTION – II

(भाग – II)

Attempt **all** *questions.*

सभी प्रश्नों का उत्तर दीजिए ।

3. (a) Consider two estimators $\hat{\theta}_1$ and $\hat{\theta}_2$.$\hat{\theta}_1$ is unbiased while $\hat{\theta}_2$ is biased. Would $\hat{\theta}_1$ always be preferred to $\hat{\theta}_2$? Why or why not ? Explain.

(b) How does an increase in the confidence level affect the precision of estimates ? Explain. (5)

(अ) दो प्राक्कलकों $\hat{\theta}_1$ व $\hat{\theta}_2$ पर विचार करें । $\hat{\theta}_1$ निष्पक्ष है, जबकि $\hat{\theta}_2$ पक्षपूर्ण है । क्या $\hat{\theta}_1$ सदैव $\hat{\theta}_2$ से उत्तम होगा ? क्यों या क्यों नहीं समझाइए ।

(ब) किस प्रकार आत्मविश्वास स्तर में वृद्धि प्राक्कलकों की सुनिश्चतता को प्रभावित करती है ? समझाइए ।

4. Suppose we want to estimate the mean price of a 2-bedroom house in Delhi. From previous studies, it is known that the standard deviation is approximately Rs. 60,000.

(i) Construct a 97% confidence interval for the mean price of a 2-bedroom house if a sample of 49 observations gave a mean of Rs. 6,00,000.

(ii) If we want to be 90% sure that the mean price of a 2-bedroom house in Delhi differs from our sample mean price by no more than Rs. 5000, how large a sample should we take ? (5)

मान लीजिए हम दिल्ली में 2-बेडरूम के घर की औसत कीमत को अनुमानित करना चाहते हैं। पूर्व अध्ययन से यह ज्ञात है कि मानक विचलन लगभग रु. 60,000/- है।

(i) एक 2-बेडरूम के घर की औसत कीमत के लिए 97% आत्मविश्वास अन्तराल बनाइए, यदि 49 इकाईयों का नमूने से माध्य रु. 6,00,000/- प्राप्त है।

(ii) यदि हम 90% सुनिश्चित होना चाहते हैं, कि दिल्ली में घर की औसत कीमत नमूना माध्य कीमत से 5000 रु. से अधिक भिन्न नहीं हो, तो हमें कितना बड़ा नमूना लेना चाहिए ?

5. (a) In 16 test runs, the gasoline consumption of an engine had a standard deviation of 2.2 gallons. Assuming normal distribution, find 99% lower confidence bound for σ^2 and interpret it.

(b) Determine the confidence level for each of the following one-sided confidence bounds :

(i) Upper bound : $\bar{x} + .74\, s/\sqrt{100}$

(ii) Lower bound : $\bar{x} - 2.06\, s/\sqrt{150}$

(iii) Upper bound : $\bar{x} + 1.753\ s/\sqrt{16}$ (5)

(अ) 16 टेस्ट रनों में, एक ईंजन के गैसोलिन खपत का मानक विचलन 2.2 गेलन है। मान लीजिए की गैसोलिन की खपत सामान्य रूप से वितरित है। σ^2 के लिए 99% निचला आत्मविश्वास बॉउंड ज्ञात करें तथा इसकी व्याख्या करें।

(ब) निम्नलिखित प्रत्येक एक-तरफा आत्मविश्वास बॉउडों के लिए आत्मविश्वास स्तर निर्धारित करें :-

(i) ऊपरी बॉउंड : $\bar{x} + .74\ s/\sqrt{100}$

(ii) निचला बॉउंड : $\bar{x} - 2.06\ s/\sqrt{150}$

(iii) ऊपरी बॉउंड : $\bar{x} + 1.753\ s/\sqrt{16}$

6. Let X_1 .... X_n be a random sample from a uniform population :

$f(x) = 1/\theta$ for $0 < x < \theta$

$= 0$ otherwise

(i) Find the moment estimator of θ.

(ii) Intuitively explain why max (X_i) will be a biased estimator of θ ?

(iii) If the bias of the estimator max (X_i) is $-\theta/(n+1)$, how would you transform it to make it unbiased ? (5)

मान लीजिए X_1..........X_n एक यूनिफार्म जनसंख्या से लिया गया एक बेतरतीब नमूना है

$f(x) = 1/\theta$ for $0 < x < \theta$

$= 0$ अन्यथा

(i) 'θ' का मोमेन्ट प्राक्कलक ज्ञात करें।

(ii) सविचार द्वारा व्याख्या करें कि क्यों $\max(X_i)$ 'θ' का पक्षधर प्राक्कलक होगा ?

(iii) यदि प्राक्कलक $\max(X_i)$ का पक्ष $-\theta/(n+1)$ है, तो आप इसे निष्पक्ष बनाने के लिए कैसे परिवर्तित करेंगे।

7. Let X_1 .... X_n be at random sample from a population with the following pdf :

$$f(x) = \frac{x^{\frac{1-\alpha}{\alpha}}}{\alpha} \quad \text{for } 0 < x < 1,\ \alpha > 1$$

$$= 0 \qquad \text{otherwise.}$$

Derive the maximum likelihood estimator of α based on a sample of size n.

(5)

मान लीजिए X_1...........X_n एक जनसंख्या से लिया गया बेतरतीब नमूना है, जिसका pdf निम्नलिखित है :-

$$f(x) = \frac{x^{\frac{1-\alpha}{\alpha}}}{\alpha} \quad \text{for } 0 < x < 1,\ \alpha > 1$$

$$= 0 \qquad \text{अन्यथा}$$

एक 'n' आकार के नमूने पर आधारित 'α' का अधिकतम लाइक्लीहुड प्राक्कलक ज्ञात करें।

SECTION – III

(भाग – III)

Question **8** *is compulsory. Do any* **two** *questions from* **9, 10** *and* **11.**

प्रश्न **8** *अनिवार्य है। प्रश्न* **9, 10** *एवं* **11** *में से किन्हीं* **दो** *का उत्तर दीजिए।*

8. (a) What do you understand by p-value ? Why is it also called the observed significance level ?

(b) Consider test of the following hypotheses :

$$H_0 : \mu = 50 \text{ against } H_a : \mu > 50$$

Using a sample of 16 items from a normal population the sample mean was found to be 51.9 and the sample standard deviation was 4. Determine the p-value of the test. Would you reject the null hypothesis at 5% level of significance ? Explain. (5)

(अ) p-value से आप क्या समझते हैं ? क्यों इसे ऑब्सजर्वड महत्व भी कहते है ?

(ब) निम्नलिखित परिकल्पना टेस्ट पर विचार करें :-

$$H_0 : \mu = 50 \text{ versus } H_a : \mu > 50$$

16 इकाईयों का नमूना एक सामान्य वितरित जनसंख्या से लिया गया और उसका नमूना माध्य 51.9 तथा मानक विचलन 4 पाया गया। टेस्ट की p-value ज्ञात करें 5 प्रतिशत महत्व स्तर पर क्या नल रद्द किया जा सकता है ? समझाइए।

9. (a) A driving test of 200 young and 400 senior drivers showed that 50% of the young and 60% of the senior drivers were cautious drivers. Use this data to test whether the proportion of cautious drivers is higher for the senior drivers. Find the p-value for the test and test at 5% level of significance. What assumptions have been made in conducting this test ?

(b) A company claims that their batteries last for at least 5.5 years. The life of such batteries are assumed to be normally distributed with standard deviation $\sigma = 9$ months. To test the claim a sample of 49 batteries is selected randomly. The claim is rejected if the sample average life of batteries is less than 5.25 years. Determine the probability of type-I error. (10)

(अ) 200 युवा व 400 वरिष्ठ चालकों के ड्राइविंग टेस्ट से यह पता चलता है, कि 50% युवा व 60% वरिष्ठ चालक सतर्क चालक हैं। यह टेस्ट करने के लिए कि सतर्क चालकों का अनुपात वरिष्ठ चालकों से अधिक है इन आंकड़ों का उपयोग करें। टेस्ट के लिए p-value ज्ञात करें और 5% महत्व स्तर पर टेस्ट करें। इस टेस्ट को करने के लिए किन मान्यताओं को लिया गया है।

(ब) एक कंपनी दावा करती है, कि उनकी बैट्रीयाँ कम से कम 5.5 वर्ष तक चलती हैं। इन सब बैट्रीयों की आयु सामान्य वितरित है वह मानक विचलन $\sigma = 9$ महीने है। इस दावे को टेस्ट करने के लिए 49 बैट्रीयों का एक बेतरतीब नमूना लिया गया। कंपनी का दावा रद्द हो जाता है अगर बैट्री की नमूना माध्य आयु 5.25 वर्ष से कम हो। टाइप-I त्रुटि की संभावना ज्ञात करें।

10. An insurance company wants to know if the average speed at which men drive cars is greater than that of women drivers. The company took a random sample of 26 cars driven by men on a highway and found the mean speed to be 72 miles per hour with a standard deviation of 2.2 miles per hour. Another independent sample of 21 cars driven by women on the same highway gave a mean speed to be 68 miles per hour with a standard deviation of 2.5 miles per hour. Assume that the speeds at which all men and all women drive cars on this highway are both normally distributed.

(i) Test at 10% level of significance whether the variance of speed of all cars driven by men on the highway is different from the variance of speed of all cars driven by women on the highway.

(ii) Based on your answer to (i) test at 5% level of significance whether the mean speed of cars driven by all men drivers on the highway is greater than the mean speed of cars driven by all women drivers. (10)

एक बीमा कम्पनी यह जानना चाहती है, कि क्या पुरूषों के गाड़ी चलाने की औसत गति महिलाओं के गाड़ी चलाने की औसत गति से अधिक है। कम्पनी ने पुरूषों के द्वारा हाइवे पर चलाई गई 26 गाड़ीयों का एक बेतरतीब नमूना लिया और पाया कि औसत गति 72 mph है और मानक विचलन 2.2 mph है। महिलाओं के द्वारा हाइवे पर चलाई गई 21 गाड़ीयों का एक और बेतरतीब नमूना लिया और पाया कि औसत गति 68 mph है और मानक विचलन 2.5 mph है। यह मान्यता है कि महिलाओं और पुरूषों के द्वारा चलाई गई गाड़ीयों की गति सामान्य वितरित है।

(i) 10 प्रतिशत महत्व स्तर पर टेस्ट करें कि क्या पुरूषों द्वारा चलाई गई गाड़ीयों की गति का विचरण महिलाओं द्वारा चलाई गई गाड़ीयों की गति के विचरण से भिन्न है।

(ii) आपके भाग (i) के उत्तर पर आधारित 5 प्रतिशत महत्व स्तर पर टेस्ट करें कि क्या पुरूषों द्वारा चलाई गईं गाड़ीयों की औसत गति महिलाओं द्वारा चलाई गई गाड़ीयों की औसत गति से अधिक है।

11. (a) Let μ denote the true average life-time of a certain type of light bulbs. Consider testing, $H_0: \mu = 6,000$ versus $H_a: \mu > 6,000$ based on a sample of size $n = 16$ from a normal population distribution with $\sigma = 150$

(i) If type I error $(\alpha) = .01$ determine the probability of making a type II error when true $\mu = 6,100$.

(ii) What should be the size of the sample if it is required that type I error $(\alpha) = .01$ and type II error, $\beta\ (6,100) = .10$?

(b) College authorities claim that at most 60% of students were residents of Delhi. A random sample of 150 students revealed that 100 were residents of Delhi. Does this data support the claim of the authorities ? Test at 10% level of significance. (10)

(अ) मान लीजिए कि 'μ' एक निश्चित प्रकार के लाइट बल्बों की वास्तविक औसत जीवन-काल को दर्शाता है। टेस्टिंग $H_0: \mu = 6,000$ व $H_a: \mu > 6,000$ पर विचार करें जो $\sigma = 150$ के साथ सामान्य जनसंख्या वितरण से लिए गए $n = 16$ आकार के नमूने पर आधारित है।

(i) जब टाइप-I त्रुटि $(\alpha) = .01$ तथा $\mu = 6100$ हो तो टाइप-II त्रुटि करने की सम्भावना को निर्धारित करें।

(ii) नमूने का आकार क्या होना चाहिए यदि आवश्यकता यह है कि जब टाइप-I त्रुटि $(\alpha) = .01$ तथा टाइप-II त्रुटि $\beta(6,100) = .10$ हो।

(ब) कॉलेज प्रशासन ह दावा करती है कि अधिकतम 60% छात्र दिल्ली के निवासी हैं। एक 150 छात्रों के बेतरतीब नमूने से यह ज्ञात हुआ कि 100 दिल्ली के निवासी हैं। क्या यह आंकड़े प्रशासन के दावों का समर्थन करते हैं ? 10% महत्व स्तर पर टेस्ट करें।

SECTION – IV

(भाग – IV)

Question 12 is compulsory. Do any **one** *question from* **13** *and* **14.**

प्रश्न **12** *अनिवार्य है। प्रश्न* **13** *एवं* **14** *में से किसी एक का उत्तर दीजिए।*

12. (a) Suppose the relation between Y and X can be described by the simple linear regression model with the true line $y = 50 - 1.5\,x$ and $\sigma = 7.5$. What is the probability that y is greater than 35 but less than 40 given that $x = 3$.

(b) Let $\mu_{Y.X} = \beta_0 + \beta_1 X + \varepsilon$ be the true relation between X and Y and its OLS estimate $\hat{\mu}_{Y.X} = b_0 + b_1 X$. Is β_0 a random variable ? Is b_0 a random variable ? Give reasons. (5)

(अ) मान लीजिए Y व X के बीच के सम्बन्ध को एक सरल रेखीय प्रतिगमन मॉडल द्वारा विस्तृत किया गया है, जिसकी वास्तविक रेखा $y = 50 - 1.5x$ व $\sigma = 7.5$ है। क्या संभावना है कि y 35 से अधिक परन्तु 40 से कम होगा यदि $x = 3$ दिया गया है।

(ब) मान लीजिए X व Y के बीच वास्तविक सम्बन्ध $\mu_{Y.X} = \beta_0 + \beta_1 X + \varepsilon$ है तथा इसका OLS प्राक्कलन $\hat{\mu}_{Y.X} = b_0 + b_1 X$ है। क्या β_0 एक बेतरतीब चर है ? क्या b_0 एक बेतरतीब चर है ? कारण बताइए।

13. For 10 countries the data for cigarette consumption per adult per year (X) and the death rates due to coronary heart disease (Y) gave the following :

$$\sum x = 160,\ \sum y = 110,\ \sum x^2 = 3154,\ \sum y^2 = 1442.54 \text{ and } \sum xy = 2042$$

(i) Estimate the ordinary least square regression line when Y is regressed on X.

(ii) Find the variance of the estimated slope coefficient (b_1).

(iii) Test $H_0: \beta_1 = 0$, $H_a: \beta_1 \neq 0$ at 1% level of significance, where β_1 is the slope of the population regression line. (10)

10 देशों के प्रति व्यस्क प्रतिवर्ष सिगरेट उपभोग (X) तथा केरानरी हृदय रोग के कारण मृत्यु दर (Y) के आंकड़े निम्नलिखित दिए गए हैं:

$\sum x = 160$, $\sum y = 110$, $\sum x^2 = 3154$, $\sum y^2 = 1442.54$ and $\sum xy = 2042$

(i) ऑर्डनरी लिईस्ट स्कार्य प्रतिगमन रेखा ज्ञात करें जब Y को X पर प्रतिशमित किया गया है।

(ii) ढलान गुणांक b_1 का विचरण ज्ञात कीजिए।

(iii) $H_0 : \beta_1 = 0, H_a : \beta_1 \neq 0$ को 1% महत्व स्तर पर टेस्ट करें। जहाँ β_1 जनसंख्या प्रतिगमन रेखा का ढलान दर्शाता है।

14. Let X : Marks obtained by a student in internal tests

Y : Marks obtained by the student in university exam

Data for 5 students shows :

X:	9	10	12	18	21
Y:	65	68	65	70	72

(i) Calculate coefficient of correlation (r).

(ii) Test at 1% level of significance, whether there is any linear relationship between the two variables.

(iii) What assumptions have you made about the joint probability distribution of X and Y to conduct the test in part (ii) ?

(iv) Calculate coefficient of determination and interpret it.

(v) Find the error sum of squares (SSE) when Y is regressed on X. (10)

मान लीजिए X : आंतरिक परीक्षा में छात्र द्वारा प्राप्त अंक।

Y : विश्वविद्यालय परीक्षा में छात्र द्वारा प्राप्त अंक।

5 विद्यार्थीयों द्वारा प्राप्त अंक।

X:	9	10	12	18	21
Y:	65	68	65	70	72

(i) सह सम्बन्ध गुणक (r) ज्ञात करें।

(ii) 1% महत्व स्तर पर ज्ञात करें, क्या दोनों चरों के बीच कोई रेखीय सम्बन्ध है।

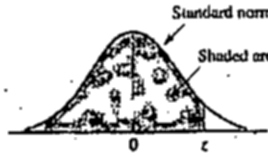
(iii) भाग (ii) में टेस्ट करने के लिए आपने X व Y के संयुक्त संभावना वितरण के लिए क्या मान्यताएँ ली हैं ?

(iv) निर्धारण का गुणांक ज्ञात करें तथा इसकी व्याख्या करें।

(v) जब Y को X पर प्रतिगमित किया जाए तो त्रुटि सम ऑफ स्कॉयर (SSE) ज्ञात करें।

Table A.3 Standard Normal Curve Areas

$\Phi(z) = P(Z \leq z)$



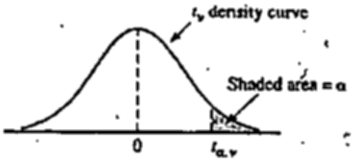
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0038
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3482
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

(continued)

Table A.3 Standard Normal Curve Areas (*cont.*) $\Phi(z) = P(Z \le z)$

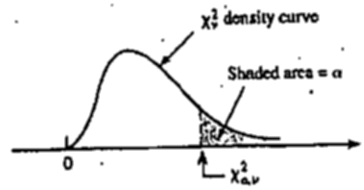
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table A.5 Critical Values for t Distributions



ν \ α	.10	.05	.025	.01	.005	.001	.0005
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
32	1.309	1.694	2.037	2.449	2.738	3.365	3.622
34	1.307	1.691	2.032	2.441	2.728	3.348	3.601
36	1.306	1.688	2.028	2.434	2.719	3.333	3.582
38	1.304	1.686	2.024	2.429	2.712	3.319	3.566
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	1.299	1.676	2.009	2.403	2.678	3.262	3.496
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

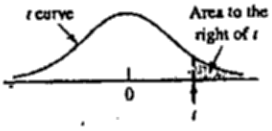
Table A.7 Critical Values for Chi-Squared Distributions



ν \ α	.995	.99	.975	.95	.90	.10	.05	.025	.01	.005
1	0.000	0.000	0.001	0.004	0.016	2.706	3.843	5.025	6.637	7.882
2	0.010	0.020	0.051	0.103	0.211	4.605	5.992	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.344	12.837
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.832	15.085	16.748
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.440	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.012	18.474	20.276
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.534	20.090	21.954
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.022	21.665	23.587
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.724	26.755
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.041	19.812	22.362	24.735	27.687	29.817
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.600	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.577	32.799
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.407	7.564	8.682	10.085	24.769	27.587	30.190	33.408	35.716
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.843	7.632	8.906	10.117	11.651	27.203	30.143	32.852	36.190	38.580
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.033	8.897	10.283	11.591	13.240	29.615	32.670	35.478	38.930	41.399
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.195	11.688	13.090	14.848	32.007	35.172	38.075	41.637	44.179
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.558
25	10.519	11.523	13.120	14.611	16.473	34.381	37.652	40.646	44.313	46.925
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.807	12.878	14.573	16.151	18.114	36.741	40.113	43.194	46.962	49.642
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.120	14.256	16.147	17.708	19.768	39.087	42.557	45.772	49.586	52.333
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
31	14.457	15.655	17.538	19.280	21.433	41.422	44.985	48.231	52.190	55.000
32	15.134	16.362	18.291	20.072	22.271	42.585	46.194	49.480	53.486	56.328
33	15.814	17.073	19.046	20.866	23.110	43.745	47.400	50.724	54.774	57.646
34	16.501	17.789	19.806	21.664	23.952	44.903	48.602	51.966	56.061	58.964
35	17.191	18.508	20.569	22.465	24.796	46.059	49.802	53.203	57.340	60.272
36	17.887	19.233	21.336	23.269	25.643	47.212	50.998	54.437	58.619	61.581
37	18.584	19.960	22.105	24.075	26.492	48.363	52.192	55.667	59.891	62.880
38	19.289	20.691	22.878	24.884	27.343	49.513	53.384	56.896	61.162	64.181
39	19.994	21.425	23.654	25.695	28.196	50.660	54.572	58.119	62.426	65.473
40	20.706	22.164	24.433	26.509	29.050	51.805	55.758	59.342	63.691	66.766

For $\nu > 40$, $\chi^2_{\alpha,\nu} \approx \nu\left(1 - \frac{2}{9\nu} + z_\alpha\sqrt{\frac{2}{9\nu}}\right)^3$

Table A.8 t Curve Tail Areas



t \ ν	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0.0	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500
0.1	.468	.465	.463	.463	.462	.462	.462	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461
0.2	.437	.430	.427	.426	.425	.424	.424	.423	.423	.423	.423	.422	.422	.422	.422	.422	.422	.422
0.3	.407	.396	.392	.390	.388	.387	.386	.386	.386	.385	.385	.385	.384	.384	.384	.384	.384	.384
0.4	.379	.364	.358	.355	.353	.352	.351	.350	.349	.349	.348	.348	.348	.347	.347	.347	.347	.347
0.5	.352	.333	.326	.322	.319	.317	.316	.315	.315	.314	.313	.313	.313	.312	.312	.312	.312	.312
0.6	.328	.305	.295	.290	.287	.285	.284	.283	.282	.281	.280	.280	.279	.279	.279	.278	.278	.278
0.7	.306	.278	.267	.261	.258	.255	.253	.252	.251	.250	.249	.249	.248	.247	.247	.247	.247	.246
0.8	.285	.254	.241	.234	.230	.227	.225	.223	.222	.221	.220	.220	.219	.218	.218	.218	.217	.217
0.9	.267	.232	.217	.210	.205	.201	.199	.197	.196	.195	.194	.193	.192	.191	.191	.191	.190	.190
1.0	.250	.211	.196	.187	.182	.178	.175	.173	.172	.170	.169	.169	.168	.167	.167	.166	.166	.165
1.1	.235	.193	.176	.167	.162	.157	.154	.152	.150	.149	.147	.146	.146	.144	.144	.144	.143	.143
1.2	.221	.177	.158	.148	.142	.138	.135	.132	.130	.129	.128	.127	.126	.124	.124	.124	.123	.123
1.3	.209	.162	.142	.132	.125	.121	.117	.115	.113	.111	.110	.109	.108	.107	.107	.106	.105	.105
1.4	.197	.148	.128	.117	.110	.106	.102	.100	.098	.096	.095	.093	.092	.091	.091	.090	.090	.089
1.5	.187	.136	.115	.104	.097	.092	.08[illegible]	.086	.084	.082	.081	.080	.079	.077	.077	.077	.076	.075
1.6	.178	.125	.104	.092	.085	.080	.077	.074	.072	.070	.069	.068	.067	.065	.065	.065	.064	.064
1.7	.169	.116	.094	.082	.075	.070	.065	.064	.062	.060	.059	.057	.056	.055	.055	.054	.054	.053
1.8	.161	.107	.085	.073	.066	.061	.057	.055	.053	.051	.050	.049	.048	.046	.046	.045	.045	.044
1.9	.154	.099	.077	.065	.058	.053	.050	.047	.045	.043	.042	.041	.040	.038	.038	.038	.037	.037
2.0	.148	.092	.070	.058	.051	.046	.043	.040	.038	.037	.035	.034	.033	.032	.032	.031	.031	.030
2.1	.141	.085	.063	.052	.045	.040	.037	.034	.033	.031	.030	.029	.028	.027	.027	.026	.025	.025
2.2	.136	.079	.058	.046	.040	.035	.032	.029	.028	.026	.025	.024	.023	.022	.022	.021	.021	.021
2.3	.131	.074	.052	.041	.035	.031	.027	.025	.023	.022	.021	.020	.019	.018	.018	.018	.017	.017
2.4	.126	.069	.048	.037	.031	.027	.024	.022	.020	.019	.018	.017	.016	.015	.015	.014	.014	.014
2.5	.121	.065	.044	.033	.027	.023	.020	.018	.017	.016	.015	.014	.013	.012	.012	.012	.011	.011
2.6	.117	.061	.040	.030	.024	.020	.018	.016	.014	.013	.012	.012	.011	.010	.010	.010	.009	.009
2.7	.113	.057	.037	.027	.021	.018	.015	.014	.012	.011	.010	.010	.009	.008	.008	.008	.008	.007
2.8	.109	.054	.034	.024	.019	.016	.013	.012	.010	.009	.009	.008	.008	.007	.007	.006	.006	.006
2.9	.106	.051	.031	.022	.017	.014	.011	.010	.009	.008	.007	.007	.006	.005	.005	.005	.005	.005
3.0	.102	.048	.029	.020	.015	.012	.010	.009	.007	.007	.006	.006	.005	.004	.004	.004	.004	.004
3.1	.099	.045	.027	.018	.013	.011	.009	.007	.006	.006	.005	.005	.004	.004	.004	.003	.003	.003
3.2	.096	.043	.025	.016	.012	.009	.008	.006	.005	.005	.004	.004	.003	.003	.003	.003	.003	.002
3.3	.094	.040	.023	.015	.011	.008	.007	.005	.005	.004	.004	.003	.003	.002	.002	.002	.002	.002
3.4	.091	.038	.021	.014	.010	.007	.006	.005	.004	.003	.003	.003	.002	.002	.002	.002	.002	.002
3.5	.089	.036	.020	.012	.009	.006	.005	.004	.003	.003	.002	.002	.002	.002	.002	.001	.001	.001
3.6	.086	.035	.018	.011	.008	.006	.004	.004	.003	.002	.002	.002	.002	.001	.001	.001	.001	.001
3.7	.084	.033	.017	.010	.007	.005	.004	.003	.002	.002	.002	.002	.001	.001	.001	.001	.001	.001
3.8	.082	.031	.016	.010	.006	.004	.003	.003	.002	.002	.001	.001	.001	.001	.001	.001	.001	.001
3.9	.080	.030	.015	.009	.006	.004	.003	.002	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001
4.0	.078	.029	.014	.008	.005	.004	.003	.002	.002	.001	.001	.001	.001	.001	.001	.001	.000	.000

(continued)

Table A.9 Critical Values for F Distributions

ν_2 = denominator df; ν_1 = numerator df

ν_2	α	1	2	3	4	5	6	7	8	9
1	.100	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86
	.050	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
	.010	4052.20	4999.50	5403.40	5624.60	5763.60	5859.00	5928.40	5981.10	6022.50
	.001	405,284	500,000	540,379	562,500	576,405	585,937	592,873	598,144	602,284
2	.100	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
	.050	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
	.010	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
	.001	998.50	999.00	999.17	999.25	999.30	999.33	999.36	999.37	999.39
3	.100	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
	.050	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
	.010	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35
	.001	167.03	148.50	141.11	137.10	134.58	132.85	131.58	130.62	129.86
4	.100	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
	.050	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
	.010	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
	.001	74.14	61.25	56.18	53.44	51.71	50.53	49.65	49.00	48.47
5	.100	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
	.050	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
	.010	16.26	13.27	12.06	11.39	10.97	10.67	10.45	10.29	10.16
	.001	47.18	37.12	33.20	31.09	29.75	28.83	28.15	27.65	27.24
6	.100	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
	.050	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
	.010	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98
	.001	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.69
7	.100	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
	.050	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
	.010	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
	.001	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.33
8	.100	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
	.050	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
	.010	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
	.001	25.41	18.49	15.83	14.39	13.48	12.86	12.40	12.05	11.77
9	.100	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
	.050	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
	.010	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
	.001	22.86	16.39	13.90	12.56	11.71	11.13	10.70	10.37	10.11
10	.100	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
	.050	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
	.010	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94
	.001	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.96
11	.100	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27
	.050	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
	.010	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63
	.001	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.35	8.12
12	.100	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
	.050	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
	.010	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
	.001	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.48

Table A.9 Critical Values for F Distributions (cont.)

ν_1 = numerator df

ν_2	α	10	12	15	20	25	30	40	50	60	120	1000
1	.100	60.19	60.71	61.22	61.74	62.05	62.26	62.53	62.69	62.79	63.06	63.30
	.050	241.88	243.91	245.95	248.01	249.26	250.10	251.14	251.77	252.20	253.25	254.19
	.010	6055.80	6106.30	6157.30	6208.70	6239.80	6260.60	6286.80	6302.50	6313.00	6339.40	6362.70
	.001	605,621	610,668	615,764	620,908	624,017	626,099	628,712	630,285	631,337	633,972	636,301
2	.100	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.47	9.48	9.49
	.050	19.40	19.41	19.43	19.45	19.46	19.46	19.47	19.48	19.48	19.49	19.49
	.010	99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.48	99.49	99.50
	.001	999.40	999.42	999.43	999.45	999.46	999.47	999.47	999.48	999.48	999.49	999.50
3	.100	5.23	5.22	5.20	5.18	5.17	5.17	5.16	5.15	5.15	5.14	5.13
	.050	8.79	8.74	8.70	8.66	8.63	8.62	8.59	8.58	8.57	8.55	8.53
	.010	27.23	27.05	26.87	26.69	26.58	26.50	26.41	26.35	26.32	26.22	26.14
	.001	129.25	128.32	127.37	126.42	125.84	125.45	124.96	124.66	124.47	123.97	123.53
4	.100	3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.80	3.79	3.78	3.76
	.050	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.70	5.69	5.66	5.63
	.010	14.55	14.37	14.20	14.02	13.91	13.84	13.75	13.69	13.65	13.56	13.47
	.001	48.05	47.41	46.76	46.10	45.70	45.43	45.09	44.88	44.75	44.40	44.09
5	.100	3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.15	3.14	3.12	3.11
	.050	4.74	4.68	4.62	4.56	4.52	4.50	4.46	4.44	4.43	4.40	4.37
	.010	10.05	9.89	9.72	9.55	9.45	9.38	9.29	9.24	9.20	9.11	9.03
	.001	26.92	26.42	25.91	25.39	25.08	24.87	24.60	24.44	24.33	24.06	23.82
6	.100	2.94	2.90	2.87	2.84	2.81	2.80	2.78	2.77	2.76	2.74	2.72
	.050	4.06	4.00	3.94	3.87	3.83	3.81	3.77	3.75	3.74	3.70	3.67
	.010	7.87	7.72	7.56	7.40	7.30	7.23	7.14	7.09	7.06	6.97	6.89
	.001	18.41	17.99	17.56	17.12	16.85	16.67	16.44	16.31	16.21	15.98	15.77
7	.100	2.70	2.67	2.63	2.59	2.57	2.56	2.54	2.52	2.51	2.49	2.47
	.050	3.64	3.57	3.51	3.44	3.40	3.38	3.34	3.32	3.30	3.27	3.23
	.010	6.62	6.47	6.31	6.16	6.06	5.99	5.91	5.86	5.82	5.74	5.66
	.001	14.08	13.71	13.32	12.93	12.69	12.53	12.33	12.20	12.12	11.91	11.72
8	.100	2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.35	2.34	2.32	2.30
	.050	3.35	3.28	3.22	3.15	3.11	3.08	3.04	3.02	3.01	2.97	2.93
	.010	5.81	5.67	5.52	5.36	5.26	5.20	5.12	5.07	5.03	4.95	4.87
	.001	11.54	11.19	10.84	10.48	10.26	10.11	9.92	9.80	9.73	9.53	9.36
9	.100	2.42	2.38	2.34	2.30	2.27	2.25	2.23	2.22	2.21	2.18	2.16
	.050	3.14	3.07	3.01	2.94	2.89	2.86	2.83	2.80	2.79	2.75	2.71
	.010	5.26	5.11	4.96	4.81	4.71	4.65	4.57	4.52	4.48	4.40	4.32
	.001	9.89	9.57	9.24	8.90	8.69	8.55	8.37	8.26	8.19	8.00	7.84
10	.100	2.32	2.28	2.24	2.20	2.17	2.16	2.13	2.12	2.11	2.08	2.06
	.050	2.98	2.91	2.85	2.77	2.73	2.70	2.66	2.64	2.62	2.58	2.54
	.010	4.85	4.71	4.56	4.41	4.31	4.25	4.17	4.12	4.08	4.00	3.92
	.001	8.75	8.45	8.13	7.80	7.60	7.47	7.30	7.19	7.12	6.94	6.78
11	.100	2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.04	2.03	2.00	1.98
	.050	2.85	2.79	2.72	2.65	2.60	2.57	2.53	2.51	2.49	2.45	2.41
	.010	4.54	4.40	4.25	4.10	4.01	3.94	3.86	3.81	3.78	3.69	3.61
	.001	7.92	7.63	7.32	7.01	6.81	6.68	6.52	6.42	6.35	6.18	6.02
12	.100	2.19	2.15	2.10	2.06	2.03	2.01	1.99	1.97	1.96	1.93	1.91
	.050	2.75	2.69	2.62	2.54	2.50	2.47	2.43	2.40	2.38	2.34	2.30
	.010	4.30	4.16	4.01	3.86	3.76	3.70	3.62	3.57	3.54	3.45	3.37
	.001	7.29	7.00	6.71	6.40	6.22	6.09	5.93	5.83	5.76	5.59	5.44

Table A.9 Critical Values for F Distributions (cont.)

ν_2 = denominator df	α	ν_1 = numerator df: 1	2	3	4	5	6	7	8	9
13	.100	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16
	.050	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
	.010	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19
	.001	17.82	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.98
14	.100	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12
	.050	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
	.010	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03
	.001	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.58
15	.100	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
	.050	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
	.010	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
	.001	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.26
16	.100	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06
	.050	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
	.010	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78
	.001	16.12	10.97	9.01	7.94	7.27	6.80	6.46	6.19	5.98
17	.100	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03
	.050	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
	.010	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68
	.001	15.72	10.66	8.73	7.68	7.02	6.56	6.22	5.96	5.75
18	.100	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00
	.050	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
	.010	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60
	.001	15.38	10.39	8.49	7.46	6.81	6.35	6.02	5.76	5.56
19	.100	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98
	.050	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
	.010	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52
	.001	15.08	10.16	8.28	7.27	6.62	6.18	5.85	5.59	5.39
20	.100	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
	.050	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
	.010	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
	.001	14.82	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24
21	.100	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95
	.050	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
	.010	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40
	.001	14.59	9.77	7.94	6.95	6.32	5.88	5.56	5.31	5.11
22	.100	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93
	.050	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
	.010	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35
	.001	14.38	9.61	7.80	6.81	6.19	5.76	5.44	5.19	4.99
23	.100	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92
	.050	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
	.010	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30
	.001	14.20	9.47	7.67	6.70	6.08	5.65	5.33	5.09	4.89
24	.100	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91
	.050	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
	.010	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26
	.001	14.03	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.80

(continued)

Table A.9 Critical Values for F Distributions (cont.)

ν_2	α	ν_1 = numerator df: 10	12	15	20	25	30	40	50	60	120	1000
13	.100	2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.92	1.90	1.88	1.85
	.050	2.67	2.60	2.53	2.46	2.41	2.38	2.34	2.31	2.30	2.25	2.21
	.010	4.10	3.96	3.82	3.66	3.57	3.51	3.43	3.38	3.34	3.25	3.18
	.001	6.80	6.52	6.23	5.93	5.75	5.63	5.47	5.37	5.30	5.14	4.99
14	.100	2.10	2.05	2.01	1.96	1.93	1.91	1.89	1.87	1.86	1.83	1.80
	.050	2.60	2.53	2.46	2.39	2.34	2.31	2.27	2.24	2.22	2.18	2.14
	.010	3.94	3.80	3.66	3.51	3.41	3.35	3.27	3.22	3.18	3.09	3.02
	.001	6.40	6.13	5.85	5.56	5.38	5.25	5.10	5.00	4.94	4.77	4.62
15	.100	2.06	2.02	1.97	1.92	1.89	1.87	1.85	1.83	1.82	1.79	1.76
	.050	2.54	2.48	2.40	2.33	2.28	2.25	2.20	2.18	2.16	2.11	2.07
	.010	3.80	3.67	3.52	3.37	3.28	3.21	3.13	3.08	3.05	2.96	2.88
	.001	6.08	5.81	5.54	5.25	5.07	4.95	4.80	4.70	4.64	4.47	4.33
16	.100	2.03	1.99	1.94	1.89	1.86	1.84	1.81	1.79	1.78	1.75	1.72
	.050	2.49	2.42	2.35	2.28	2.23	2.19	2.15	2.12	2.11	2.06	2.02
	.010	3.69	3.55	3.41	3.26	3.16	3.10	3.02	2.97	2.93	2.84	2.76
	.001	5.81	5.55	5.27	4.99	4.82	4.70	4.54	4.45	4.39	4.23	4.08
17	.100	2.00	1.96	1.91	1.86	1.83	1.81	1.78	1.76	1.75	1.72	1.69
	.050	2.45	2.38	2.31	2.23	2.18	2.15	2.10	2.08	2.06	2.01	1.97
	.010	3.59	3.46	3.31	3.16	3.07	3.00	2.92	2.87	2.83	2.75	2.66
	.001	5.58	5.32	5.05	4.78	4.60	4.48	4.33	4.24	4.18	4.02	3.87
18	.100	1.98	1.93	1.89	1.84	1.80	1.78	1.75	1.74	1.72	1.69	1.66
	.050	2.41	2.34	2.27	2.19	2.14	2.11	2.06	2.04	2.02	1.97	1.92
	.010	3.51	3.37	3.23	3.08	2.98	2.92	2.84	2.78	2.75	2.66	2.58
	.001	5.39	5.13	4.87	4.59	4.42	4.30	4.15	4.06	4.00	3.84	3.69
19	.100	1.96	1.91	1.86	1.81	1.78	1.76	1.73	1.71	1.70	1.67	1.64
	.050	2.38	2.31	2.23	2.16	2.11	2.07	2.03	2.00	1.98	1.93	1.88
	.010	3.43	3.30	3.15	3.00	2.91	2.84	2.76	2.71	2.67	2.58	2.50
	.001	5.22	4.97	4.70	4.43	4.26	4.14	3.99	3.90	3.84	3.68	3.53
20	.100	1.94	1.89	1.84	1.79	1.76	1.74	1.71	1.69	1.68	1.64	1.61
	.050	2.35	2.28	2.20	2.12	2.07	2.04	1.99	1.97	1.95	1.90	1.85
	.010	3.37	3.23	3.09	2.94	2.84	2.78	2.69	2.64	2.61	2.52	2.43
	.001	5.08	4.82	4.56	4.29	4.12	4.00	3.86	3.77	3.70	3.54	3.40
21	.100	1.92	1.87	1.83	1.78	1.74	1.72	1.69	1.67	1.66	1.62	1.59
	.050	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.94	1.92	1.87	1.82
	.010	3.31	3.17	3.03	2.88	2.79	2.72	2.64	2.58	2.55	2.46	2.37
	.001	4.95	4.70	4.44	4.17	4.00	3.88	3.74	3.64	3.58	3.42	3.28
22	.100	1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.65	1.64	1.60	1.57
	.050	2.30	2.23	2.15	2.07	2.02	1.98	1.94	1.91	1.89	1.84	1.79
	.010	3.26	3.12	2.98	2.83	2.73	2.67	2.58	2.53	2.50	2.40	2.32
	.001	4.83	4.58	4.33	4.06	3.89	3.78	3.63	3.54	3.48	3.32	3.17
23	.100	1.89	1.84	1.80	1.74	1.71	1.69	1.66	1.64	1.62	1.59	1.55
	.050	2.27	2.20	2.13	2.05	2.00	1.96	1.91	1.88	1.86	1.81	1.76
	.010	3.21	3.07	2.93	2.78	2.69	2.62	2.54	2.48	2.45	2.35	2.27
	.001	4.73	4.48	4.23	3.96	3.79	3.68	3.53	3.44	3.38	3.22	3.08
24	.100	1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.62	1.61	1.57	1.54
	.050	2.25	2.18	2.11	2.03	1.97	1.94	1.89	1.86	1.84	1.79	1.74
	.010	3.17	3.03	2.89	2.74	2.64	2.58	2.49	2.44	2.40	2.31	2.22
	.001	4.64	4.39	4.14	3.87	3.71	3.59	3.45	3.36	3.29	3.14	2.99

(continued)

Table A.9 Critical Values for F Distributions (cont.)

ν_2 = denominator df	α	ν_1 = numerator df: 1	2	3	4	5	6	7	8	9
25	.100	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89
	.050	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
	.010	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22
	.001	13.88	9.22	7.45	6.49	5.89	5.46	5.15	4.91	4.71
26	.100	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88
	.050	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
	.010	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18
	.001	13.74	9.12	7.36	6.41	5.80	5.38	5.07	4.83	4.64
27	.100	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87
	.050	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
	.010	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15
	.001	13.61	9.02	7.27	6.33	5.73	5.31	5.00	4.76	4.57
28	.100	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87
	.050	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
	.010	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12
	.001	13.50	8.93	7.19	6.25	5.66	5.24	4.93	4.69	4.50
29	.100	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86
	.050	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
	.010	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09
	.001	13.39	8.85	7.12	6.19	5.59	5.18	4.87	4.64	4.45
30	.100	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85
	.050	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
	.010	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
	.001	13.29	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39
40	.100	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79
	.050	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
	.010	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89
	.001	12.61	8.25	6.59	5.70	5.13	4.73	4.44	4.21	4.02
50	.100	2.81	2.41	2.20	2.06	1.97	1.90	1.84	1.80	1.76
	.050	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07
	.010	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78
	.001	12.22	7.96	6.34	5.46	4.90	4.51	4.22	4.00	3.82
60	.100	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
	.050	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
	.010	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
	.001	11.97	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69
100	.100	2.76	2.36	2.14	2.00	1.91	1.83	1.78	1.73	1.69
	.050	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97
	.010	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59
	.001	11.50	7.41	5.86	5.02	4.48	4.11	3.83	3.61	3.44
200	.100	2.73	2.33	2.11	1.97	1.88	1.80	1.75	1.70	1.66
	.050	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93
	.010	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50
	.001	11.15	7.15	5.63	4.81	4.29	3.92	3.65	3.43	3.26
1000	.100	2.71	2.31	2.09	1.95	1.85	1.78	1.72	1.68	1.64
	.050	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89
	.010	6.66	4.63	3.80	3.34	3.04	2.82	2.66	2.53	2.43
	.001	10.89	6.96	5.46	4.65	4.14	3.78	3.51	3.30	3.13

(continued)

Table A.9 Critical Values for F Distributions (cont.)

ν_2	α	ν_1 = numerator df: 10	12	15	20	25	30	40	50	60	120	1000
25	.100	1.87	1.82	1.77	1.72	1.68	1.66	1.63	1.61	1.59	1.56	1.52
	.050	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.84	1.82	1.77	1.72
	.010	3.13	2.99	2.85	2.70	2.60	2.54	2.45	2.40	2.36	2.27	2.18
	.001	4.56	4.31	4.06	3.79	3.63	3.52	3.37	3.28	3.22	3.06	2.91
26	.100	1.86	1.81	1.76	1.71	1.67	1.65	1.61	1.59	1.58	1.54	1.51
	.050	2.22	2.15	2.07	1.99	1.94	1.90	1.85	1.82	1.80	1.75	1.70
	.010	3.09	2.96	2.81	2.66	2.57	2.50	2.42	2.36	2.33	2.23	2.14
	.001	4.48	4.24	3.99	3.72	3.56	3.44	3.30	3.21	3.15	2.99	2.84
27	.100	1.85	1.80	1.75	1.70	1.66	1.64	1.60	1.58	1.57	1.53	1.50
	.050	2.20	2.13	2.06	1.97	1.92	1.88	1.84	1.81	1.79	1.73	1.68
	.010	3.06	2.93	2.78	2.63	2.54	2.47	2.38	2.33	2.29	2.20	2.11
	.001	4.41	4.17	3.92	3.66	3.49	3.38	3.23	3.14	3.08	2.92	2.78
28	.100	1.84	1.79	1.74	1.69	1.65	1.63	1.59	1.57	1.56	1.52	1.48
	.050	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.79	1.77	1.71	1.66
	.010	3.03	2.90	2.75	2.60	2.51	2.44	2.35	2.30	2.26	2.17	2.08
	.001	4.35	4.11	3.86	3.60	3.43	3.32	3.18	3.09	3.02	2.86	2.72
29	.100	1.83	1.78	1.73	1.68	1.64	1.62	1.58	1.56	1.55	1.51	1.47
	.050	2.18	2.10	2.03	1.94	1.89	1.85	1.81	1.77	1.75	1.70	1.65
	.010	3.00	2.87	2.73	2.57	2.48	2.41	2.33	2.27	2.23	2.14	2.05
	.001	4.29	4.05	3.80	3.54	3.38	3.27	3.12	3.03	2.97	2.81	2.66
30	.100	1.82	1.77	1.72	1.67	1.63	1.61	1.57	1.55	1.54	1.50	1.46
	.050	2.16	2.09	2.01	1.93	1.88	1.84	1.79	1.76	1.74	1.68	1.63
	.010	2.98	2.84	2.70	2.55	2.45	2.39	2.30	2.25	2.21	2.11	2.02
	.001	4.24	4.00	3.75	3.49	3.33	3.22	3.07	2.98	2.92	2.76	2.61
40	.100	1.76	1.71	1.66	1.61	1.57	1.54	1.51	1.48	1.47	1.42	1.38
	.050	2.08	2.00	1.92	1.84	1.78	1.74	1.69	1.66	1.64	1.58	1.52
	.010	2.80	2.66	2.52	2.37	2.27	2.20	2.11	2.06	2.02	1.92	1.82
	.001	3.87	3.64	3.40	3.14	2.98	2.87	2.73	2.64	2.57	2.41	2.25
50	.100	1.73	1.68	1.63	1.57	1.53	1.50	1.46	1.44	1.42	1.38	1.33
	.050	2.03	1.95	1.87	1.78	1.73	1.69	1.63	1.60	1.58	1.51	1.45
	.010	2.70	2.56	2.42	2.27	2.17	2.10	2.01	1.95	1.91	1.80	1.70
	.001	3.67	3.44	3.20	2.95	2.79	2.68	2.53	2.44	2.38	2.21	2.05
60	.100	1.71	1.66	1.60	1.54	1.50	1.48	1.44	1.41	1.40	1.35	1.30
	.050	1.99	1.92	1.84	1.75	1.69	1.65	1.59	1.56	1.53	1.47	1.40
	.010	2.63	2.50	2.35	2.20	2.10	2.03	1.94	1.88	1.84	1.73	1.62
	.001	3.54	3.32	3.08	2.83	2.67	2.55	2.41	2.32	2.25	2.08	1.92
100	.100	1.66	1.61	1.56	1.49	1.45	1.42	1.38	1.35	1.34	1.28	1.22
	.050	1.93	1.85	1.77	1.68	1.62	1.57	1.52	1.48	1.45	1.38	1.30
	.010	2.50	2.37	2.22	2.07	1.97	1.89	1.80	1.74	1.69	1.57	1.45
	.001	3.30	3.07	2.84	2.59	2.43	2.32	2.17	2.08	2.01	1.83	1.64
200	.100	1.63	1.58	1.52	1.46	1.41	1.38	1.34	1.31	1.29	1.23	1.16
	.050	1.88	1.80	1.72	1.62	1.56	1.52	1.46	1.41	1.39	1.30	1.21
	.010	2.41	2.27	2.13	1.97	1.87	1.79	1.69	1.63	1.58	1.45	1.30
	.001	3.12	2.90	2.67	2.42	2.26	2.15	2.00	1.90	1.83	1.64	1.43
1000	.100	1.61	1.55	1.49	1.43	1.38	1.35	1.30	1.27	1.25	1.18	1.08
	.050	1.84	1.76	1.68	1.58	1.52	1.47	1.41	1.36	1.33	1.24	1.11
	.010	2.34	2.20	2.06	1.90	1.79	1.72	1.61	1.54	1.50	1.35	1.16
	.001	2.99	2.77	2.54	2.30	2.14	2.02	1.87	1.77	1.69	1.49	1.22