This question paper contains 7 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **7041**

Unique Paper Code : **227506** **D**

Name of the Paper : **Topics in Microeconomics-I**

Name of the Course : **B.A. (Hons.) Economics**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions carry equal marks.

Do *three* questions from Part A and *two* from Part B.

Candidates are allowed to use simple calculators.

Part A

1. Consider the Bertrand duopoly game where the cost function of Firm 1 is $C_1(q_1) = 10q_1$ and the cost function of Firm 2 is $C_2(q_2) = 15q_2$. Firms simultaneously choose the prices they charge. Assume that firms can choose only non-negative integers as prices. The market demand function is $D(P) = 100 - P$ for $P < a$ and zero otherwise. The firm that charges

a lower price captures the entire market. Further suppose that when both firms charge the same price, each firm serves half the market demand. Find all the pure strategy Nash equilibria of this game.

If the cost functions of both firms change to $C_i(q_i) = 5q_i$, for all q_i, $i = 1, 2$ (firms can choose only non-negative integers as prices in this case also), find the new set of Nash equilibria. 9+6=15

There are 3 bidders named Bidder 1, Bidder 2 and Bidder 3 with valuations equal to 30, 20 and 10 respectively in a second-price sealed-bid auction for a single object. Assume that valuations of bidders are common knowledge. Bids can be any non-negative real numbers and each bidder submits a sealed bid without knowing the bids submitted by others. The bidder who submits a bid higher than the bid submitted by the other two bidders gets the object at a price equal to the highest bid submitted by the other bidders and gets utility equal to her/his valuation minus the price paid. Other (unsuccessful) bidders don't pay anything and each one of them gets a payoff of zero. Also assume that if more than one bidder submits the highest bid, the player with the highest valuation amongst those whose bids are the highest gets the object. Give necessary and sufficient conditions for a pure strategy profile to be Nash equilibrium in this game. Is there any Nash equilibrium in which no player uses a weakly dominated strategy ? 9+6=15

3. Countries A and B are at war. Navy of country A has only one submarine with which it can target one (and only one) of three shipping convoys of country B. Country B has no Navy, but has a single helicopter armed with anti-submarine missiles that can be assigned to one of the convoys. The value of convoy k to country B is v_k, with $v_1 > v_2 > v_3 > 0$. Navy of country A can sink a convoy, if the convoy is not defended by the anti-submarine helicopter and the submarine of country A attacks it. Countries A wants to maximize the expected value of damage to Country B and country B wants to minimize it. In particular, if the submarine targets convoy 'i', the payoffs of Countries A and B are v_i and $-v_i$ respectively in case the helicopter is assigned to some other convoy 'j'. If the submarine targets the convoy to which the helicopter is assigned, each country gets zero payoff. Formulate the situation as a simultaneous move strategic game and show that there are no pure strategy Nash equilibria. Find the mixed strategy Nash equilibria. You need to consider two cases (depending on whether $(v_1 + v_2)\, v_3 > v_1 v_2$ or not). 5+10=15

4. Consider the following version of Hotelling's model of electoral competition. There are three potential political candidates. Each one of them has to simultaneously decide whether or not

to enter a political contest. If a candidate decides to contest, she/he also has to simultaneously choose a policy position, which is modelled as choosing a number in some interval $[a, b]$, without knowing what the other candidates have decided. There is a continuum of voters, each of whom has a favourite position; the distribution of favourite positions is given by cumulative probability distribution function F. Interpret $F(x)$ as the proportion of voters whose favourite policy position is less than or equal to x. Assume that F is strictly increasing and continuous. A candidate attracts the votes of those citizens whose favourite positions are closer to her/his position than to the position of any other candidate. If two or more candidates take the same position, they equally split the votes that the position attracts. Each potential candidate prefers to be the sole winning candidate than to tie for first place with others, prefers to tie for first place than to stay out of the electoral race, and prefers to stay out of the race than to enter and lose. Formulate this situation as a strategic game and show that there is no Nash equilibrium in pure strategies when there are three potential candidates and F is continuous. 4+11=15

Part B

5. Consider the following version of Stackleberg's duopoly model. There are two firms in an industry. Firm 1 moves first and chooses a non-negative quantity q_1. Firm 2 observes q_1 and then decides its level of output q_2, where $q_2 \geq 0$. The price 'P' at which each firm's output is sold is given by the inverse demand function :

$$P = 100 - Q, \text{ for } Q \leq 100 \text{ and zero otherwise (where } Q = q_1 + q_2)$$

The total cost function of firm i for $i = 1, 2$ is :

$$C_i(q_i) = 400, \text{ if } q_i > 0$$

and

$$C_i(q_i) = 0, \text{ if } q_i = 0.$$

(Marginal cost is zero, the only cost is fixed cost that is incurred if a strictly positive quantity is produced)

Assume that each firm maximizes its profit. Formulate this situation as an extensive game and find its subgame perfect equilibria. Briefly comment on the economic interpretation of your results. 3+10+2=15

6. Consider the following version of the Centipede game. The game starts in period 1 with two Rupees on the table, which player 1 can grab or leave on the table. If player 1 grabs the money, the game terminates with payoffs equal to 2 and 0 to players 1 and 2 respectively (i.e., the amounts of money they obtain in the game). If Player 1 leaves the money on the table, the game continues to period 2 and the money on the table increases by 2 Rupees to 4 Rupees. In period 2 it is the turn of Player 2 to move. She/he can grab the money or leave it on the table. If Player 2 leaves the money on the table, the amount of money on the table again increases by 2 Rupees to 6 Rupees and it is the turn of Player 1 to move, who can grab the money or leave it on the table. The game continues like this upto period 100, with players alternating (Player 1 moves in odd periods and Player 2 in even periods) and the pile of money increases by 2 Rupees every period, if no player has grabbed the money in any previous period. The game terminates whenever a player grabs the money, say in period k, for some $k \leq 100$. The player who grabs in period k gets all the money on the table ($2k$ Rupees) and the other player gets nothing. In period 100 player 2 can either grab 200 Rupees on the table (Player 1 in that case gets a payoff equal to 0) or leave the money on the table, in which case the game terminates and each player gets a payoff of 100. Formulate this situation as a game and find its subgame perfect equilibria. Are there any Nash equilibria that are not subgame perfect ? Is the set of outcomes in Nash equilibria different from the set of outcomes in subgame perfect equilibria ? 2+6+5+2=15

7. Two players take turns removing stones from a pile of n stones. Player 1 moves first. Each player has to remove one or two stones on each of his turns. The player who removes the last stone is the winner and gets Rs. 100 from the other player. Formulate this as an extensive game and show that player 1 is the winner in any subgame perfect, equilibrium of the game if $n = 2k + 1$ or $2k + 2$, whereas player 2 is the winner if $n = 3k$ for any non-negative integer k. (Start with the case $n = 1$, 2 and 3 and then generalize using induction.) 15