[This question paper contains 6 printed pages.]

Sr. No. of Question Paper: 6013

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Your Roll No.....

Unique Paper Code

: 227506

Name of the Paper

: Topics in Microeconomics - I

Name of the Course

: B.A. (Hons.) Economics

Semester

: V

Duration: 3 Hours

Maximum Marks: 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

- 2. There are seven questions in all. Answer any three questions from Part A and any two from Part B.
- 3. Candidates are allowed to use simple calculators.

PART A

(Answer any three questions from this part)

1. (a) Find all the pure strategy and mixed strategy Nash equilibria of the following two-person game:

| · · · · · · · · · · · · · · · · · · · | Pînyer 2 | | | |
|---------------------------------------|----------|------|--------|-------|
| | | Left | Centre | Right |
| Player 1 | Top | 4, 5 | 1,6 | 5, 6 |
| | Middle | 3, 5 | 2, 5 | 5, 4 |
| | Bottom | 2,0 | 0, 1 | 0, 3 |

- (b) Consider the Bertrand duopoly game with constant unit cost "c" for both firms and the demand function at the price p taking the specific form D(p) = a p for $p \le a$, and zero otherwise. Show that the strategy where a firm chooses the price "c" is weakly dominated by a price \bar{p} where $c < \bar{p} < a$. Assume that both firms share the market equally if they charge the same price. (10, 5)
- 2. There are N students who visit a café. At the café, each student would like to approach and win the attention of a famous (but temperamental) rock star who also happens to be there. The catch is that if two or more students approach the rock star, the latter gets annoyed and leaves the café. In the game, each student simultaneously and independently selects an action from the set {Stay away, Approach}. For each student i, the payoff equals:
 - 0, if the student decides to stay away from the rock star, regardless of the actions of the other players;
 - 5, if the student approaches the rock star, and if every other student j (j \neq : i) stays away, and
 - -10, if the student approaches the rock star, and if some other student j (j $i \neq i$) also does the same.
 - (a) Find the pure strategy Nash equilibrium/equilibria of this game.
 - (b) Suppose each student randomizes between the actions "Stay away" and "Approach" with probabilities p and (1-p), respectively. Solve for the mixed strategy Nash equilibrium?
 - (c) Suppose it is only x students who randomize between the two actions $(2 \le x \le N)$, and all students who randomize assign the same positive probability p to the action "Stay away". Construct a Nash equilibrium where x students randomize and the remaining N-x students choose "Stay away" with certainty.

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3. (a) Consider the Cournot oligopoly model where there are n firms that choose outputs simultaneously, The inverse demand function takes the form:

$$P(Q) = \begin{cases} 5 - Q & \text{if } Q \le 5 \\ 0 & \text{if } Q > 5 \end{cases}$$

where Q is the firms' total output. The cost function of each firm i is $C_i(q_i) = 2q^i$ for all q_i (i = 1, 2, ..., n), and is common knowledge. Assume that each firm maximizes its profit. Solve for the pure strategy Nash equilibrium.

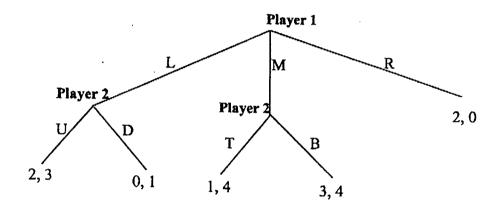
- (b) Consider an all-pay sealed-bid auction with two bidders. Bidders can submit any non-negative bids for the object being auctioned. The valuations of the two bidders are v_1 and v_2 , respectively, with $v_1 > v_2 > 0$. Assume that these valuations are common knowledge. The person who bids higher wins the object. Ties are broken in favour of the bidder with the higher valuation namely, bidder 1. Each bidder, regardless of whether he or she wins the object, has to pay the auctioneer the lower of the two bids. Find all the pure strategy Nash equilibria of this game. (5, 10)
- 4. Consider a population of voters uniformly distributed along the ideological spectrum from left (x = 0) to right (x = 1). Each of the candidates contesting for a single office simultaneously chooses a campaign platform that is, a point on the line between x = 0 and x = 1. The voters observe the candidates' choices, and then each voter votes for the candidate whose platform is closest to the voter's position on the spectrum. If, for example, there are two candidates, and they choose x₁ = 0.3 and x₂ = 0.6, then all the voters to the left of x = 0.45 vote for candidate 1, all those to the right vote for candidate 2, and candidate 2 wins the election with 55% of the vote. Suppose that the candidates care only about being elected they do not care about victory margins or ideological positions. Further assume that candidates who choose the same platform equally split the votes cast for that platform. Moreover, in case of a tie among leading vote-getters, each candidate is equally likely to get elected.

- (a) If there are two candidates, find the unique pure strategy Nash equilibrium?
- (b) If there are three candidates, show that one possible Nash equilibrium is where candidate i chooses x = 1/3, while j and k choose x = 2/3 (i, j, k = 1, 2, 3).

PART B

(Answer any two questions from this part)

5. (a) For the following extensive-form game:



- (i) Find all the pure strategy Nash equilibria.
- (ii) Which of these Nash equilibria are subgame perfect? Explain your answer.
- (b) Two people use the following procedure to split Rs. 1000 Person 1 first proposes an amount x ($0 \le x \le 1000$) which he would like to keep for himself. If person 2 accepts, person 1 gets to keep Rs. x and person 2 receives Rs. (1000 x). If person 2 rejects the allocation, then person 2 gets 0 but player 1 still gets what he proposed (that is, Rs. x). Find the subgame perfect equilibrium/equilibria of this game. Find the subgame perfect equilibrium/equilibria when person 2 does not have the option of rejecting the allocation that person 1 suggests. (8,7)

6. (a) Two distinct policies A and B are being considered for adoption by the Central government (Centre) and the States using the following mechanism - First, the States decide whether to maintain status quo (in which case the game ends), or send a proposal to the centre for passing either policy A or policy B or both. The Centre on receiving the proposal decides whether to accept or reject the policy or policies being proposed. If the Centre rejects the Stafes' proposal, the status quo prevails. Model this as an extensive game and find the subgame perfect equilibrium/equilibria. Assume that the rankings of the States and the Centre over the four possible outcomes are as follows, where a higher number represents a more favoured outcome.

| Outcome | States | Centre |
|-------------------------------|--------|--------|
| A is accepted | 4 | 1 |
| B is accepted | 1 | 4 |
| Both A and B are accepted | 3 | 3 |
| Neither (status quo prevails) | 2 | 2 |

- (b) Suppose the rules of the game in part (a) are changed in only one respect if the States put forth a proposal for adopting both A and B, the Centre may now choose to accept both policies, or reject both, or reject one of the two policies contained in it. Model this situation as an extensive game and find the subgame perfect equilibrium/equilibria.
- (c) Compare the equilibrium outcomes for parts (a) and (b), and comment whether the Centre is necessarily better off having more options available to it. (15)
- 7. (a) Two players take turns removing stones from a pile of n stones. Player 1 moves first. Each player, on each of her turns, removes either one stone or two stones from the pile. The person who removes the last stone(s) loses the game. Formulate this as an extensive game and find the winner in each subgame perfect equilibrium for any arbitrary value of n (Start with the case n = 1, 2, and 3 and then generalize using induction).

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(b) Consider the Stackelberg duopoly model. There are two firms in an industry. Firm 1 moves first and chooses a non-negative quantity q_1 . Firm 2 observes q_1 and then decides the level of output q_2 , where $q_2 \ge 0$. The price "P" at which each firm's output is sold is given by the inverse demand function:

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$$P(Q) = \begin{cases} 1000 - Q & \text{if } Q \le 1000 \\ 0 & \text{if } Q > 1000 \end{cases}, (Q = q_1 + q_2)$$

The cost function of each firm i is $C_i(q_i) = 100q_i$, (i = 1, 2). Assume that each firm maximizes its profit. Formulate this situation as an extensive game and find its subgame perfect equilibrium/equilibria. (10,5)