This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8148**

Unique Paper Code : **235685** **D**

Name of the Paper : **Elements of Analysis**

Name of the Course : **B.A. (Hons.) Economics**

Semester : **VI**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *three* questions from Section I,

any *two* questions from Section II.

and any *two* questions from Section III.

Section I

1. (*a*) State order completeness property and show that the set of rational numbers is not order complete. 5

(*b*) Show that for all real numbers x and y : 5

$$\frac{|x+y|}{1+|x+y|} \leq \frac{|x|}{1+|x|} + \frac{|y|}{1+|y|}$$

2. (*a*) Show that every Cauchy sequence is bounded but the converse is not true. 5

(*b*) Show that the sequence $\langle a_n \rangle$, where

$$a_n = 1 + \frac{1}{5} + \frac{1}{5^2} + ... + \frac{1}{5^{n-1}}$$

converges. Find $\lim_{n\to\infty} a_n$. 5

3. (*a*) Prove that if $r > 1$, then : 5

$$\lim_{n\to\infty} r^n = +\infty.$$

(*b*) State Cauchy's first theorem on limits and show that : 5

$$\lim_{n\to\infty} \left[\frac{1}{n^2} + \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + ... + \frac{1}{(2n)^2} \right] = 0.$$

4. (*a*) State Cauchy's general principle of convergence and show that the sequence $\langle a_n \rangle$ defined by : 5

$$a_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}$$

does not converge.

(*b*) Define $\langle a_n \rangle$ as :

$$a_1 = 1, a_{n+1} = \frac{3 + 2a_n}{2 + a_n}, \forall\ n \geq 1$$

Show that $\langle a_n \rangle$ is convergent. Also find its limit. 5

Section II

5. (*a*) If the series $\sum_{n=1}^{\infty} a_n$ converges, then show that :

$$\lim_{n\to\infty} a_n = 0.$$

Also show by an example, that the converse is not true. 6

(*b*) Show that the series

$$\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)^{1/n}$$

does not converge. 6½

6. (*a*) Test for convergence the series 6

$$1 + \frac{1}{2!} + \frac{1}{3!} + .. + \frac{1}{n!} + ...$$

(*b*) Test for convergence the series 6½

$$\frac{1}{\sqrt{1.2}} + \frac{1}{\sqrt{2.3}} + \frac{1}{\sqrt{3.4}} + ...$$

7. (*a*) Test for convergence the series whose nth term is : 6

$$\frac{n^{n^2}}{(n+1)^{n^2}}$$

(*b*) State Leibnitz test. Test the convergence of the series : 6½

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$$

Section III

8. (*a*) If the power series $\sum_{n=1}^{\infty} a_n x^n$ is such that $a_n \neq 0$ for all n and : 5

$$\lim_{n \to \infty} \left(\frac{a_{n+1}}{a_n} \right) = \frac{1}{R},$$

then $\sum_{n=1}^{\infty} a_n x^n$ is convergent for $|x| < R$ and divergent for $|x| > R$.

(*b*) Find the radius of convegence of the power series : 5

$$x + \frac{x^2}{2^2} + \frac{2!}{3^3} x^3 + \frac{3!}{4^4} x^4 + \ldots$$

9. (*a*) Find the domain of convergence of the series : 5

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{2^n (3n-1)}.$$

(*b*) Define term by term integration theorem of power series $\sum_{n=1}^{\infty} a_n x^n$. Show that the series $\sum_{n=1}^{\infty} a_n x^n$ and its integrated power series $\sum_{n=1}^{\infty} \frac{a_n}{n+1} x^{n+1}$ have the same radius of convergence. 5

10. (*a*) Define exponential function $E(x)$ as a sum of a power series. Show that the domain is the set of all real numbers. Prove that :

$$E(x + y) = E(x) E(y) \quad \forall\, x, y \in R.$$

If e denotes $E(1)$, prove that $E(x) = e^x$ for all real x. 5

(*b*) Define cosine and sine function as sum of power series and prove that : 5

$$C(x + y) = C(x) C(y) - S(x) S(y)$$

where S and C denotes sine and cosine respectively.