This question paper contains 4+2 printed pages]

Your Roll No. ....

9651

B.A./B.Sc. (Hons.)/I **B**

MATHEMATICS—Unit I

(Vector, Calculus and Geometry)

(Admissions of 2008 and before)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *one* question from each Section.

Marks are indicated against each question.

Section 1

1. (*a*) Find the first and second derivatives of :

$$\left[\vec{r}\ \ \frac{d\vec{r}}{dt}\ \ \frac{d^2\vec{r}}{dt^2}\right],$$

where the rectangular bracket has usual meaning. 3

(*b*) A particle moves along the curve :

$$x = e^{-t}, \quad y = 2\cos(3t), \quad z = 2\sin(3t).$$

ind the magnitudes of acceleration and velocity when time t is zero. 3

(*c*) Prove that :

$$\operatorname{div}(\vec{V}_1 \times \vec{V}_2) = \vec{V}_2 . \operatorname{curl} \vec{V}_1 - \vec{V}_2 . \operatorname{curl} \vec{V}_2 \cdot$$ 3½

2. (*a*) Find the constants a, b, c so that :

$$\vec{F} = (x+2y+az)i + (bx-3y-z)j + (4x+cy+2z)k$$

is irrotational. 3

(*b*) The position vector of a point at time t is given by :

$$\vec{r} = e^{t}(i\cos t + j\sin t)$$

show that $\vec{a} = 2\vec{v} - 2\vec{r}$, where $\vec{a}$, $\vec{v}$ are respectively acceleration and velocity of the particle. 3

(c) Prove that for any two differentiable vectors $\vec{F}$ and $\vec{G}$:

$$\vec{\nabla} \times (\vec{F} \times \vec{G}) = \vec{F}\,(\vec{\nabla} . \vec{G}) - \vec{G}\,(\vec{\nabla} . \vec{F}) + (\vec{G} . \vec{\nabla})\,\vec{F} - (\vec{F} . \vec{\nabla})\,\vec{G}$$ 3½

Section 2

3. (a) Prove that the two circles, which pass through the two points (0, a) and (0, $-a$) and touch the straight line $y = mx + c$ will cut orthogonally if $c^2 = a^2 (2 + m^2)$. 4½

(b) (i) If e and e' be the eccentricities of a hyperbola and its conjugate, prove that :

$$\frac{1}{e^2} + \frac{1}{e'^2} = 1.$$ 2½

(ii) Find the locus of the point of intersection of tangents to

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

which meet at right angles. 2½

4. (*a*) If the tangents to a parabola $y^2 = 4ax$ at two points P and Q meet in T, then prove that :

(*i*) TP and TQ subtend equal angles at the focus S.

(*ii*) $(ST)^2 = SP \cdot SQ$. 5

(*b*) (*i*) Prove that the sum of the squares of the perpendiculars on any tangent to the ellipse :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

from two points on the minor axis each distant $\sqrt{a^2 - b^2}$ from the centre is $2a^2$. 2½

(*ii*) Prove that a common tangent to the two circles of a coaxal system subtends a right angle at either limiting point of the system. 2

Section 3

5. Trace the central conic :

$$2x^2 + y^2 - 2xy + 2x - 2y = 0 .$$ 9½

6. Trace the central conic :

$$9x^2 - 32xy + 9y^2 + 60x + 10y = 64\frac{1}{2} .$$ 9½

Section 4

7. (*a*) A variable plane passes through a fixed point (*a*, *b*, *c*) and cuts the coordinate axes in the points A, B, C. Show that the locus of the centre of the sphere OABC is :

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2 .$$ 4½

(*b*) Show that the general equation of a cone which touches the three coordinate planes is :

$$\sqrt{fx} \pm \sqrt{gy} \pm \sqrt{hz} = 0 ;$$

f, *g*, *h* being parameters. 5

8. (*a*) Obtain the equation of the right circular cylinder described on the circle through the three points :

$$(1, 0, 0), (0, 1, 0), (0, 0, 1)$$

as guiding circle. 4½

(*b*) Find the condition that the plane $lx + my + nz = 0$ should touch the cone :

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0.$$ 5