This question paper contains 7 printed pages]

Your Roll No. ....

9652

B.A./B.Sc. (Hons.)/I **B**

MATHEMATICS—Unit-II

(Algebra-I)

(Admissions of 2008 and before)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

1. (*a*) If

$$\sin\alpha + \sin\beta + \sin\gamma = \cos\alpha + \cos\beta + \cos\gamma = 0,$$

prove that :

(*i*) $\cos(\beta+\gamma) + \cos(\gamma+\alpha) + \cos(\alpha+\beta)$

$$= \sin(\beta+\gamma) + \sin(\gamma+\alpha) + \sin(\alpha+\beta) = 0.$$

(ii) $\sin 2\alpha + \sin 2\beta + \sin 2\gamma$

$$= \cos 2\alpha + \cos 2\beta + \cos 2\gamma = 0.$$

(iii) $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma$

$$= \sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = \frac{3}{2}.$$ 4½

(b) (i) Find all the values of $\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{3/4}$ and show that the continued product of all these values is 1. 2

(ii) If

$$(a_1 + ib_1)(a_2 + ib_2)..........(a_n + ib_n) = A + iB,$$

prove that :

$$\tan^{-1}\frac{b_1}{a_1} + \tan^{-1}\frac{b_2}{a_2} + ...$$

$$+ \tan^{-1}\frac{b_n}{a_n} = \tan^{-1}\frac{B}{A}$$

and

$$(a_1^2 + b_1^2)(a_2^2 + b_2^2)........(a_n^2 + b_n^2) = A^2 + B^2.$$ 2½

(c) Sum the series :

$$\sum_{\alpha=1}^{n} \sqrt{1+\sin \alpha x}.$$ 4½

2. (a) (i) Prove that an elementary column operation on the product of two matrices is equivalent to the same elementary column operation on the post factor. 3½

(ii) Prove that every skew-symmetric matrix of odd order is singular. 1½

(b) Define rank of a matrix. 2

Show that the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are collinear iff the rank of the matrix :

$$\begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix}$$

is less than 3. 3

(c) (i) If λ is a characteristic root of a non-singular matrix A, then prove that $\frac{|A|}{\lambda}$ is a characteristic root of the matrix adj A. 2½

(ii) If A is a Skew-Hermitian matrix, show that its characteristic roots are either purely imaginaries or zero. 2½

3. (a) (i) When a system of linear equations AX = B is consistent, where A is $m \times n$, X is $n \times 1$ and B is $m \times 1$ matrices ? 1½

(ii) For what values of λ does the following system of equations have a solution :

$$x + y + z = 1$$

$$x + 2y + 4z = \lambda$$

$$x + 4y + 10z = \lambda^2$$

and also find the solution in each case. 3½ (*b*) (*i*) Form the cubic whose roots are the values of α, β, γ given by the relations :

$$\sum \alpha = 3, \ \sum \alpha^2 = 5, \ \sum \alpha^3 = 11.$$

Hence find the value of $\sum \alpha^4$. 2½

(*ii*) Solve the equation :

$$x^3 + x^2 - 16x + 20 = 0,$$

being given that some of its roots are repeated. 2½

(*c*) If α, β, γ be the roots of the equation :

$$x^3 - px^2 + qx - r = 0, \ r \neq 0,$$

find the equation whose roots are :

$$\beta\gamma + \frac{1}{\alpha}, \ \gamma\alpha + \frac{1}{\beta}, \ \alpha\beta + \frac{1}{\gamma}.$$ 3

Also find the values of

$$\sum (\alpha-\beta)^2 \text{ and } \sum \frac{\beta^2+\gamma^2}{\beta\gamma}$$

from the given equation. 2

4. (*a*) (*i*) Find g.c.d. of 578 and 442. Also find two integers m and n such that :

$$\text{g.c.d } (578, 442) = m \cdot 578 + n \cdot 442.$$ 2½

(*ii*) If g.c.d. $(a, b) = 1$, then show that :

$$\text{g.c.d. } (a+b, a^2-ab+b^2) = 1 \text{ or } 3.$$ 2

(*b*) (*i*) If $ac \equiv cb \pmod{m}$ and g.c.d. $(c, m) = 1$, then show that $a \equiv b \pmod{m}$. 3

(*ii*) Find the remainder when 41^{65} is divided by 7. 1½ (*c*) (*i*) If σ is a k-cycle, show that σ is an odd permutation if k is even and is an even permutation if k is odd. 2

(*ii*) Find all the four cycles of S_4.

(*iii*) Let

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 1 & 2 & & & 7 & 8 & 9 & 6 \end{pmatrix}$$

be an even permutation. Find $\sigma(4)$ and $\sigma(5)$. 1½