[This question paper contains 3 printed pages.]

1465-A *Your Roll No.* ....

B.A./B.Sc. (Hons.)/II **A**

MATHEMATICS – Unit VI

(Differential Equations – I)

(Admissions of 2008 and before)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

All *questions are compulsory.*

SECTION I

1. Solve any **two** of the following :

(a) $(2x^3y^2+4x^2y+2xy^2+xy^4+2y)dx + 2(y^3+x^2y+x)dy = 0$ (3)

(b) $(px-y)(py+x) = h^2p$ (3)

(c) $p^3 - (x^2+xy+y^2)p^2 + (x^3y+x^2y^2+xy^3)p - x^3y^3 = 0$ (3)

2. In a certain decay problem, initially there is 'm' mg of the material present. After 2 years it is observed that 5 percent of the material has decayed. Find an expression for the mass present at any time t. Also find time taken for 10% of the original mass to decay. (3)

P.T.O.

SECTION II

3. Solve any **two** of the following :

(a) $(D^2 - 4D + 4)y = 8x^2e^{2x}\sin 2x$, where $D \equiv \dfrac{d}{dx}$. (3)

(b) $(x^4D^4 + 6x^3D^3 + 9x^2D^2 + 3xD + 1)y = (1 + \log x)^2$;

where $D \equiv \dfrac{d}{dx}$. (3)

(c) $4y^{(2)} - 4y^{(1)} + y = e^x + 2\cos 2x$

by the method of undetermined co-efficients. (3)

4. Show that the Wronskian of two solutions of the differential equation

$$a_0(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = 0,$$

where $a_0(x) \neq 0 \ \forall \ x$ and $a_0(x)$, $a_1(x)$ & $a_2(x)$ are its f^{ns} of x on [a, b]; is either identically zero or never zero on (a, b). (4)

SECTION III

5. Find power series solution of any **two** of the following :

(a) $(2+x^2)y^{(2)} + xy^{(1)} - (1+x)y = 0$ about $x_0 = 0$ (5)

(b) $9x(1-x)y^{(2)} - 12y^{(1)} + 4y = 0$ about $x_0 = 0$ (5)

(c) $2xy^{(2)} + (x+1)y^{(1)} + 3y = 0$ about $x_0 = 0$ (5)

SECTION IV

6. Solve any **two** of the following :

(a) $4x_1 + 9y_1 + 11x + 31y = e^t$

$3x_1 + 7y_1 + 8x + 24y = e^{2t}$; where

$$x_1 = \frac{dx}{dt}$$

$$y_1 = \frac{dy}{dt}$$ (3)

(b) $$\frac{dx}{x^2 - yz} = \frac{dy}{y^2 - zx} = \frac{dz}{z^2 - xy}$$ (3)

(c) $z(z + y^2)dx + z(z + x^2)dy - xy(x + y)dz = 0$ (3)

7. Using Picard's method find upto three successive approximations, the solution of the differential equation

$$\frac{dy}{dx} = 2y - 2x^2 - 3$$

$$y(0) = 2$$ (3)