[This question paper contains 4 printed pages.]

1469-A *Your Roll No.* ....

B.A./B.Sc. (Hons.)/II **A**

MATHEMATICS – Unit X

(Probability and Mathematical Statistics)

(Admissions of 2008 and before)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **two** *parts from each Section.*
Marks are indicated against each question.

SECTION – I

1. (a) (i) Define probability function. (2)

(ii) If $A_1, A_2, ---- A_n$ is a finite sequence of mutually exclusive events, then prove that

$$P(A_1 \cup A_2 \cup --- \cup A_n) = P(A_1) + P(A_2) + --- + P(A_n)$$

(2½)

(b) If n letters are placed at random in n correctly addressed envelopes, find the probability that none of the letters are placed incorrectly addressed envelopes. (4½)

P.T.O.

(c) A coin is tossed until a head shows up. Set X be the number of trials required.

Find (a) the probability function of X.

(b) the moment generating function of X.

(c) the mean and variance. (4½)

SECTION - II

2. (a) If X is a Poisson's variate with parameter λ, then show that $E(|X-1|) = 2e^{-\lambda} + \lambda - 1$. (5)

(b) Prove the recurrence relation for moments of Binomial distribution B(n, p) :

$$\mu_{r+1} = pq\left[n r \mu_{r-1} + \frac{d\mu_r}{dp}\right]$$

Hence find μ_2, μ_3, μ_4. (5)

(c) If X is a Binomial variate with parameter n and p, then

$$P(X \geq k) = \frac{1}{\beta(k, n-k+1)} \int_0^p u^{k-1} (1-u)^{n-k} du$$ (5)

SECTION - III

3. (a) Consider the experiment of tossing two tetrahedra marked 1, 2, 3, 4.

Let X : score of first tetrahedron

Y : the greater of the two scores.

(i) Obtain the joint distribution of X and Y.

(ii) Obtain the Marginal distribution of X and Y.

(iii) Find $P(X \geq 3,\ Y \geq 2)$.

(iv) Obtain the conditional distribution of X, given $Y = 2$.

(v) Are X and Y independent ? (5)

(b) Let $f(x, y) = k(x + y) I_{(0,1)}^{(x)}\ I_{(0,1)}^{(y)}$

Find (i) the value of k

(ii) $P\left(0 < X < \frac{1}{2},\ 0 < Y < \frac{1}{4}\right)$

(iii) $f_{\frac{Y}{X}}\left(\frac{y}{x}\right)$

(iv) Conditional cumulative distribution function of Y given $X = x$. (5)

(c) If X and Y are jointly distributed continuous random variables, prove that

$$E\left(E\left[\frac{g(Y)}{X}\right]\right) = E(g(Y))$$ (5)

SECTION - IV

4. (a) For a normal distribution $N(\mu, \sigma)$ if $\mu_r' = E(x^r)$, prove that

$$\mu'_{r+2} = 2\,\mu\,\mu'_{r+1} + \left(\sigma^2 - \mu^2\right)\mu'_r + \sigma^3\,\frac{d\mu'_r}{d\sigma} \qquad (5)$$

(b) State 'Weak Law of Large Numbers'. Let X_i assume the value i^α and $-i^\alpha$ with equal probabilities. Prove that the sequence $\langle X_n \rangle$ of independent random variables satisfies weak law of large numbers if $\alpha < \frac{1}{2}$. (5)

(c) Define characteristic function ϕ_X for any random variable X and prove that if the probability density function f_X is an even function, then the characteristic function ϕ_X is an even real valued function. (5)