This question paper contains 4 printed pages.]

9656 *Your Roll No. ....*

B.A. / B.Sc. (Hons.) / II **B**

MATHEMATICS – Unit VII

(Mechanics – I)

(Admissions of 2008 and before)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

***All** sections are compulsory.*

*Attempt any **two** parts from each section.*

SECTION - I

1. (a) Three forces P, Q, R act along the sides of the triangle formed by the lines

$x + y = 1, y - x = 1, y = 2.$

Find the equation of the line of action of their resultant. 5

(b) A force of magnitude P, acting up and along a smooth inclined plane, can support a weight W. When acting horizontally, it can support a weight

w. Find a relation between P, W and w, not involving the inclination of the plane. 5

(c) Six equal uniform rods **AB, BC, CD, DE, EF** and **FA,** each of weight W, are freely jointed at their extremities so as to form a hexagon. The rod **AB** is fixed in a horizontal position and the middle points of **AB** and **DE** are joined by a string. Find the tension in the string when the hexagon is in equilibrium in a vertical plane. 5

SECTION - II

2. (a) State and prove Pappus' Theorem for the mass centre of a uniform distribution along a plane curve. Hence find the mass centre of a uniform semicircular wire. 5

(b) A uniform rod AB rests inside a smooth parabola with its axis vertical and vertex downwards. Find the angle θ that AB makes with the horizontal, and determine whether the equilibrium is stable. 5

(c) A uniform ladder of weight W rests on a rough horizontal ground and against a smooth vertical wall, the ladder being inclined at an angle A to the horizontal. Prove that a horizontal force P applied at the foot of the ladder to make it move towards the wall must be at least

$$W\left(\mu+\frac{\cot A}{2}\right)$$

where μ is the coefficient of friction between the ladder and the floor. 5

SECTION - III

3. (a) A force (4, 3, –2) acts at (1, 0, 3) and another force (2, –3, 5) acts at (–3, 1, 2).

Determine (i) the equation of the central axis and (ii) the pitch of the equipollent wrench. 5

(b) Prove that in an infinitesimal rotation $\delta\mathbf{n}$ of a rigid body about a point O, the displacement of a point P of the body, whose position vector with respect to O is $\mathbf{r}$, is given by

$$\delta\mathbf{r} = \delta\mathbf{n} \times \mathbf{r}$$ 5

(c) Define generalised coordinates and generalised forces for a system of particles. Find a set of generalised forces for a particle constrained to move on the surface of a smooth sphere. 5

SECTION - IV

4. (a) If the line joining two points A and B is vertical and lies wholly within a homogeneous fluid in equilibrium, prove that the difference in pressure at A and B is proportional to the difference in depth. 4

(b) Prove that the depth of the centre of pressure of a triangular lamina whose vertices are at depths α, β and γ is given by

$$\frac{(\alpha^2+\beta^2+\gamma^2+\alpha\beta+\beta\gamma+\gamma\alpha}{2(\alpha+\beta+\gamma)}$$ 4

(c) The centre of pressure of a parallelogram completely immersed in a heavy homogeneous fluid lies in one of the diagonals. Prove that the other diagonal is horizontal. 4