This question paper contains 4 printed pages.]

9659 *Your Roll No. ....*

B.A. / B.Sc. (Hons.) / II **B**

MATHEMATICS – Unit X

(Probability and Mathematical Statistics)

(Admissions of 2008 and before)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt two parts from each question.

Section I

1. (a) State and prove Baye's theorem. 4½

(b) Five per cent of the people have high blood pressure. Of the people with high blood pressure, 75 per cent drink alcohol; whereas, only 50 per cent of the people without high blood pressure drink alcohol. What per cent of the drinkers have high blood pressure ? 4½

(c) Let $f(x) = Ke^{-\alpha x}(1-e^{-\alpha x})I_{(O\alpha)}(x)$

(i) Find K such that $f(.)$ is a density function.

(ii) Find the corresponding c.d.f.

(iii) Find $P[X > 1]$. 4½

Section II

2. (a) For a binomial distribution, show that

$$u_{r+1} = pq\left\{nr\mu_{r-1} + \frac{d\mu_r}{dp}\right\}$$

and deduce the values of μ_2, μ_3 and μ_4. 5

(b) If X has beta distribution, Can $E\left(\frac{1}{X}\right)$ be equal to unity ? 5

(c) If X has gamma distribution with parameters r and λ ($r \in N$), then

$$F_x(x) = 1 - \sum_{j=0}^{r-1} \frac{e^{-\lambda x}(\lambda x)^j}{j!}$$ 5

Section III

3. (a) Suppose X_1 and X_2 are independent random variables with distribution given by

$$P[X_i = -1] = P[X_i = 1] = \frac{1}{2} \text{ for } i = 1, 2.$$

check if X_1 and $X_1 X_2$ independent ? 5

(b) Prove

$$F_X(x)+F_Y(y)-1 \le F_{X,Y}(x,y) \le \sqrt{F_X(x)F_Y(y)}$$

for all x, y. 5

(c) Three fair coins are tossed. Let X denote the number of heads on the first two coins, and let Y denote the number of tails on the last two coins.

(i) Find the joint distribution of X and Y.

(ii) Find the conditional distribution of Y given that X = 1.

(iii) Find Cov [X, Y]. 5

Section IV

4. (a) Suppose that an instructor assumes that a student's final score is the value of a normally distributed random variable. If the instructor decides to award a grade of A to those students whose score exceeds $\mu + \sigma$, a B to those students whose score falls between μ and $\mu + \sigma$, a C if a score falls between $\mu - \sigma$ and μ, a D if a score falls between $\mu - 2\sigma$ and $\mu - \sigma$, and an F if the score falls below $\mu - 2\sigma$. Find the proportions of each grade. 4½

(b) Let $\{x_n\}$ be a sequence of independent random variables such that

$$P\left(X_n = \frac{1}{\sqrt{n}} \right) = p_n,$$

$$P\left(X_n = 1 + \frac{1}{\sqrt{n}} \right) = 1 - p_n$$

Examine whether the weak law of large numbers holds true for this sequence. 4½

(c) State and prove the central limit theorem for independent and identically distributed random variables for which the moment generating function exists. 4½