Your Roll No.....

Sl. No. of Question Paper: 615

Unique Paper Code : 235483

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Name of Course : B.A. (Hons.)

Name of Paper : Mathematics-I (Algebra & Calculus) Other than Economics

Semester : IV

Duration : 3 hours **Maximum Marks : 75**

Instructions for Candidates:

Question No. 1 is compulsory and carries 15 marks. Attempt six more questions selecting at least two questions from each section. Each question carries 10 marks.

1. (i) If $A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ and $B=\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}$, where $i^2=-1$. Verify that $(A+B)^2=A^2+B^2$.

(ii) Evaluate $\lim_{x\to 4}\frac{\sqrt{x+5}-3}{(x-4)}$.

(iii) Find $\frac{dy}{dx}$, when $y=x^2\log(3x+7)$

(iv) Show that $\begin{vmatrix}y+z & x & y\\ z+x & z & x\\ x+y & y & z\end{vmatrix}=(x+y+z)(x-y)^2$.

(v) Evaluate $\int x^3\sin(x^4+1)dx$

SECTION-I

2. (i) Find the inverse of the matrix $A=\begin{bmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{bmatrix}$ and verify tha

$A^{-1}A=I_3$, where I_3 is the identity matrix of order 3.

(ii) Solve the following system of equations by Cramer's rule :

$x-4y-z=11,\ 2x-5y+2z=39,\ -3x+2y+z=1$

3. (i) Find the equation of parabola, whose focus is (3, –4) and directrix is the line $x+y-2=0$.

(ii) If $A=\begin{bmatrix}2 & 1 & 3\\ 2 & 0 & 1\\ -4 & 5 & 6\end{bmatrix}$, verify that $A(\text{adj } A) = |A| I_3$, where I_3 is the identity matrix of order 3.

4. (i) Find the centre and radius of the following circle :
$x^2+y^2+2x+17y+4=0$

(ii) Find the co-ordinates of the foci, the vertices, the length of major and minor axes and the eccentricity of the ellipse $x^2 + 4y^2 = 16$. Also sketch the graph of the ellipse.

SECTION –II

5. (i) If $x^y = e^{x-y}$ show that $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$.

(ii) Examine the continuity of the function

$$f(x)=\begin{cases}\dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}, & \text{when } x\neq 0\\ 0, & \text{when } x=0\end{cases} \quad \text{at } x=0.$$

6. (i) Find the relative maxima and minima of the function $f(x)=x^4-8x^3+22x^2-24x+1$.

(ii) Determine the intervals of concavity and points of inflection of the curve $y=x^4-4x^3-18x^2+1$.

7. (i) Verify Rolle's theorem for $f(x)=x^3-6x^2+11x-6$, in $[1,3]$.

(ii) Write down the Maclaurin series expansion for the function $f(x)=e^x$.

SECTION III

8. (i) Find the area of the region bounded by the curves $y = x^2$ and $y = x$.

(ii) Evaluate $\int \frac{x^3 - 5x}{(x^2 - 9)(x^2 + 1)} dx$.

9. (i) Find the general solution of the differential equation

$$(1 + y)xy\frac{dy}{dx} = (1 - x^2)(1 - y)$$

(ii) The marginal cost function of a firm is MC = $(\log x)^2$. Find the total cost function when the cost of producing one unit is Rs 20.

10. (i) Evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$.

(ii) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{2^n}$