[This question paper contains 3 printed pages.]

Your Roll No. ....

9669 **B**

B.A./B.Sc. (Hons.)/III

MATHEMATICS—Paper XVII and XVIII (*iv*)

(Integral Transforms and Boundary Value Problems)

Time : 2 *Hours* *Maximum Marks* : 38

(*Write your Roll No. on the top immediately on receipt of this question paper.*)

Answer any ***two*** parts from each question.

The symbols used have their usual meaning.

1. (*a*) Evaluate :

(*i*) $L\left(\frac{\sin 3t}{t}\right)$.

(*ii*) $L^{-1}\left(\frac{e^{-2s}}{s^2+1}\right)$.

(*b*) State the Convolution theorem for inverse Laplace transforms. Hence, evaluate :

$$L^{-1}\left(\frac{f(s)}{s}\right),$$

Where $L^{-1}(f(s)) = F(t)$.

(*c*) By using the Laplace transform technique, solve the following initial value problem :

$y''(t) + 2y'(t) + y(t) = 3te^{-t}$

$y(0) = 4,\ y'(0) = 2.$ 4½, 4½, 4½.

[P. T. O.

2. (*a*) Find the Fourier series corresponding to the function :

$$f(x) = \begin{cases} x+1, & -1 < x < 0, \\ x-1, & 0 < x < 1. \end{cases}$$

(*b*) Solve the following boundary value problem for the transverse displacements in a string :

$y_{tt}(x, t) = a^2 y_{xx}(x, t),\ 0 < x < 1,\ t > 0,$

$y(0, t) = 0;\ y(1, t) = 0$

$y(x, 0) = A \sin \pi x,$

$y_t(x, 0) = 0,$

Where A is a constant.

(*c*) A string, stretched between the fixed points 0 and π on the x-axis, is initially straight with prescribed distribution of velocities $y_t(x, 0) = \sin x$. Write the boundary value problem in $y(x, t)$ and solve it.

5,5,5

3. (*a*) A slab occupies the region $0 \le x \le C$. There is a constant flux of heat $\phi = \phi_0$ into the slab through the face $x = 0$. The face $x = C$ is kept at temperature $u = 0$. Set up the boundary value problem for the steady-state temperature $u(x)$ in the slab and solve it.

(*b*) Solve the following boundary value problem for the temperature $u(x, t)$ in an infinite insulated slab of material bounded by the planes $x = 0$ and $x = \pi$:

$u_t(x, t) = k\ u_{xx}(x, t),\ 0 \le x \le \pi,\ t > 0,$

$u_x(0, t) = 0,\ u_x(\pi, t) = 0,\ t > 0,$

$u(x, 0) = 2 - \cos x,\ 0 < x < \pi.$

(*c*) Solve the following boundary value problem for the temperature $u(x, t)$ in an infinite slab of material :

$u_t(x, t) = k\ u_{xx}(x, t),\ 0 < x < \pi,\ t > 0,$

$u(0, t) = 0,\ u(\pi, t) = u_0$

$u(x, 0) = 0,$

Where u_0 is a constant. 5,5,5

4. (*a*) State and prove the Fourier integral theorem.

(*b*) Find the Fourier cosine integral representation of the function :

$f(x) = e^{-bx},\ x > 0,\ b > 0.$

(*c*) Find the Fourier cosine transform of the function :

$$f(x) = \begin{cases} x & , 0 < x < 1/2, \\ 1-x & , 1/2 < x < 1, \\ 0 & , x > 1. \end{cases}$$

Also write the inverse transform. 4½,4½,4½