This question paper contains 4+1 printed pages]

Your Roll No. .....

9671

B.A./B.Sc. (Hons.)/III **B**

MATHEMATICS—Paper XVII and XVIII (vi)

(Linear Programming)

Time : 2 *Hours* *Maximum Marks* : 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

All questions are compulsory.

1. (*a*) If an optimal basic solution is non-degenerate and $z_j - c_j > 0$ for every vector not in the basis, show that optimal solution is unique. 4½

(*b*) Define a convex set.

Test the convexity of the set :

$X = \{(x_1, x_2) | x_1 x_2 \leq 1, x_1 \geq 0, x_2 \geq 0\}$ 4½

P.T.O.

(*c*) For a given system of simultaneous linear equations in n unknowns $Ax = b \{m < n\}$ $x^T \in R^n$, where A is an $m \times n$ matrix of rank m, define a basic feasible solution. Reduce $2a_1 + 4a_2 + a_3 = b$ to a basic feasible solution if $a_1 = [2, 1]$, $a_2 = [-1, 4]$, $a_3 = [2, 0]$ 4½

2. (*a*) Using simplex method solve the system of equations $Ax = b$, where :

$$A = \begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Also find A^{-1}. 4½

(*b*) Show that the following linear programming problem has no feasible solution :

Maximize : $z = x_1 + 4x_2 + 3x_3$

Subject to :

$$2x_1 - x_2 + 5x_3 = 40$$

$$x_1 + 2x_2 - 3x_3 \geq 22$$

$$3x_1 + x_2 + 2x_3 = 30$$

$$x_1, x_2, x_3 \geq 0$$ 4½

(*c*) Find alternate optimal solution of the Linear Programming Problem :

Maximize : $z = x_1 + 2x_2 + 3x_3$

Subject to :

$$x_1 + 2x_2 + 3x_3 \leq 10$$

$$x_1 + x_2 \leq 5$$

$$x_1 \leq 1$$

$$x_1, x_2 \geq 0$$ 4½

3. (*a*) Write the dual of the following problem so that the dual variables are all non-negative and the requirement vector is non-negative :

Minimize : $z = 2x_1 + 3x_2 - 5x_3$

Subject to :

$$x_1 + x_2 - x_3 + x_4 \geq 5$$

$$2x_1 + x_3 \leq 4$$

$$x_2 + x_3 + x_4 = 6$$

$x_1, x_2, x_3 \geq 0$, x_4 unrestricted in sign. 5

(*b*) Obtain an optimum basic feasible solution to the following transportation problem :

Warehouse

Factory	W_1	W_2	W_3	W_4	Capacity
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Requirements	5	8	7	14	5

(*c*) Solve the following assignment problem : 5

	I	II	III	IV	V
1	11	17	8	16	20
2	9	7	12	6	15
3	13	16	15	12	16
4	21	24	17	28	26
5	14	10	12	11	15

4. (*a*) Solve the following game by linear programming : 5

Player B

$$\textbf{Player A} \quad \begin{bmatrix} -1 & 1 & 1 \\ 2 & -2 & 2 \\ 3 & 3 & -3 \end{bmatrix}$$

(*b*) Solve the following game graphically : 5

Player B

$$\textbf{Player A} \quad \begin{bmatrix} 3 & -3 & 4 \\ -1 & 1 & -3 \end{bmatrix}$$

(*c*) Use the relation of dominance to solve the game whose pay-off matrix A is given by : 5

Player B

$$\textbf{Player A} \quad \begin{bmatrix} 4 & 3 & 1 & 2 & 2 & 2 \\ 4 & 3 & 7 & -5 & 1 & 2 \\ 4 & 3 & 4 & -1 & 2 & 2 \\ 4 & 3 & 3 & -2 & 2 & 2 \end{bmatrix}$$