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Your Roll No. ....

9660

B.A./B.Sc. (Hons.)/III **B**

MATHEMATICS – Unit XI

(Differential Equations–II)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *four* questions in all,

selecting *one* question from each Section.

Section I

1\. (*a*) Explain Lagrange's method of solving a linear partial differential equation of the first order. Hence, solve the differential equation :

$$(y + zx)p - (x + yz)q = y^2 - x^2.$$ 5

(*b*) Find the complete integral of the equation :

$$xp + 3yq = 2(z - x^2q^2).$$ 4½

2. (*a*) Find the condition of compatibility of first order partial differential equations

$$f(x, y, z, p, q) = 0 \text{ and } g(x, y, z, p, q) = 0.$$ 3½

(*b*) Find the complete integral of the equation :

$$p^2q^2 + x^2y^2 = x^2q^2(x^2 + y^2).$$ 3

(*c*) Using Jacobi's method, find the complete integral of the partial differential equation :

$$p_1p_2p_3 = z^3x_1x_2x_3.$$ 3

Section II

3. (*a*) Reduce the equation :

$$x^2r - y^2t + px - qy = x^2$$

to canonical form and hence find its general solution. 4½ (*b*) Find the solution for the non-homogeneous wave equation :

$$\frac{\partial^2 z}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 z}{\partial y^2} + f(x, y) = 0$$

with the initial conditions :

$z(x, 0) = \phi(x)$

$zy(x, 0) = \psi(x).$ 5

4. (*a*) Prove that, for the equation :

$$\frac{\partial^2 z}{\partial x \partial y} + \frac{z}{4} = 0,$$

the Green's function is

$$w(x, y, \xi, \eta) = J_0\left(\sqrt{(x-\xi)(y-\eta)}\right)$$

where $J_0(z)$ denotes Bessel's function of the first kind of order zero. 5

(b) Reduce the equation :

$$y^2 \frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$$

to canonical form. $4\frac{1}{2}$

Section III

5. (a) Show that in cylindrical co-ordinates ρ, z, ϕ, Laplace's equation has solution of the form $R(\rho)\ e^{\pm mz \pm in\phi}$, where $R(\rho)$ is a solution of Bessel's equation

$$\frac{d^2R}{d\rho^2} + \frac{1}{\rho}\frac{dR}{d\rho} + \left(m^2 - \frac{n^2}{\rho^2}\right)R = 0.$$

If $R \to 0$ as $z \to \infty$ and is finite when $\rho = 0$, show that, in the usual notations for Bessel's functions, the appropriate solutions are made up of terms of the form $J_n(m\rho)\ e^{-mz \pm in\phi}$. 5

(*b*) Solve one-dimensional diffusion equation

$$\frac{\partial^2\theta}{\partial x^2} = \frac{1}{k}\frac{\partial\theta}{\partial t},$$

in the range $0 \le x \le 2\pi$, $t \ge 0$ subject to the boundary conditions :

$$\theta(x, 0) = \sin^3 x \quad \text{for } 0 \le x \le 2\pi$$

$$\theta(0, t) = \theta(2\pi, t) = 0 \text{ for } t \ge 0.$$ 4½

6. (*a*) Solve three-dimensional wave equation in spherical polar co-ordinates by the method of separation of variables. 5

(*b*) Determine the temperature $\theta(\rho, t)$ in the infinite cylinder $0 \le \rho \le a$, when the initial temperature is $\theta(\rho, 0) = f(\rho)$ and the surface $\rho = a$ is maintained at zero temperature. 4½

Section IV

7. (*a*) Solve by Monge's method :

$$xy(t - r) + (x^2 - y^2)(s - 2) = py - qx.$$ 3½

(*b*) Define reducible and irreducible linear partial differential equations with constant coefficients. 3

(*c*) Solve :

$$(3D^2 - 2D'^2 + D - 1)z = 4e^{x+y}\cos(x + y).$$ 3

8. (*a*) Solve by Monge's method :

$$y^2r + 2xys + x^2t + px + qy = 0.$$ 3½

(*b*) Solve the equation :

$$(x^2D^2 - 4y^2D'^2 - 4yD' - 1)z = x^2y^2 \log y.$$ 3

(*c*) Solve the equation :

$$(3D^2 - 2D'^2 + D - 1)z = 4e^{x+y} \cos(x + y).$$ 3