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Your Roll No. ....

9668

B.A./B.Sc. (Hons.)/III **B**

MATHEMATICS—Paper XVII & XVIII (*iii*)

(Discrete Mathematics)

Time : 2 *Hours* *Maximum Marks* : 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

1. (*a*) Attempt any *two* parts :

(*i*) Determine if there exists a graph whose degree sequence is :

5, 4, 3, 2, 5, 1

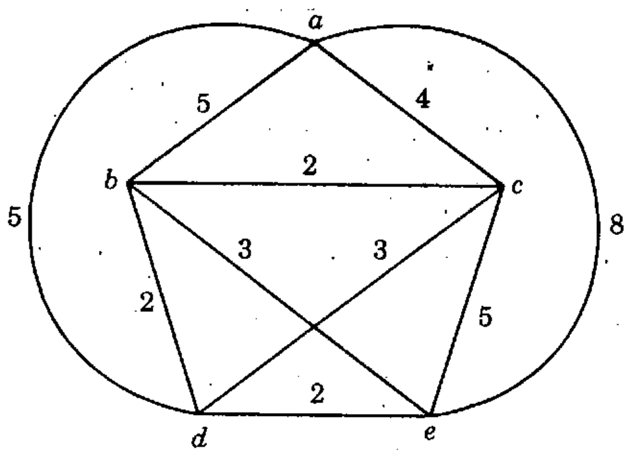
if exists, draw a graph, otherwise explain why no graph exists.

(*ii*) Find a formula for the number of edges in K_n.

(*iii*) Show that for $n \geq 5$, K_n cannot be planar.

2½+2½=5

(*b*) Use the nearest-neighbour method to determine a Hamiltonian circuit for the graph given below, starting at vertex *a* :



Can you determine a minimum Hamiltonian circuit ? Justify your answer. 4½

2. (*a*) Attempt any *two* parts :

(*i*) For the FSM whose state transition and the associated output as shown below, determine the output sequence corresponding to the input sequence 110101 :

State	Input		Output
	0	1	
⇒ A	B	C	1
B	B	C	1
C	A	C	0
D	B	D	0

Also, determine an input sequence that will produce the output sequence 110101.

(*ii*) Design a FSM that accepts all binary sequences that end with the digits 011.

(*iii*) Define K-equivalent states. Determine all the 1-equivalent states of the following FSM :

State	Input		Output
	0	1	
⇒ A	H	C	0
B	G	B	0
C	A	B	0
D	D	C	0
E	H	B	0
F	D	E	1
G	H	C	1
H	A	E	0

2½+2½=5

(*b*) A three-state finite state machine has {0, 1} as its input and output alphabets. Given the following input sequence and its corresponding output sequence, determine the machine :

Input sequence : 0 0 0 1 0 1 0 1

Output sequence : 0 1 1 0 0 1 1 1 0 4½

3. (*a*) Attempt any *two* parts :

(*i*) Let *a, b* be numeric function such that :

$$a_r = \begin{cases} 1 & \text{if} \quad r = 0 \\ 2 & \text{if} \quad r = 1 \\ 0 & \text{if} \quad r \geq 2 \end{cases} \text{ and } b_r = (-2)^r, \ (r \geq 0)$$

Determine $c = a * b$.

(*ii*) Let *a, b* and *c* be three-numeric functions :

$$a_r = 3r - 2, \quad r \geq 0$$

$$b_r = \begin{cases} \dfrac{2}{r} + 7 & , \quad r > 0 \\ 0 & , \quad \text{else} \end{cases} \text{ and}$$

$$c_r = \begin{cases} r \ln r & , \quad r > 0 \\ 0 & , \quad \text{else} \end{cases}$$

Does a dominates *b* asymptotically ?

Does a dominates *c* asymptotically ?

(*iii*) Let A be the incidence matrix of the block design (b, v, r, k, λ) configuration. Show that :

$$A'A = (r - \lambda) I + \lambda J$$

where the symbols have their usual meaning. (I is the unit matrix and J is the matrix with every entry equal to 1.) 2½+2½=5

(*b*) Define a Hadamard matrix. Show how 16 × 16 Hadamard matrix can be obtained from a known 2 × 2 Hadamard matrix and deduce a (15, 7, 3) configuration. 4½

4. (*a*) Attempt any *two* parts :

(*i*) Determine the discrete numeric function corresponding to the following generating function :

$$A(z) = \frac{z^4}{1 - 5z + 6z^2}.$$

(*ii*) Obtain the generating function of the numeric function :

$(0^2; 1^2, 2^2, 3^2, ..., r^2, ...)$.

(*iii*) Solve the recurrence relation :

$a_r + 3a_{r-1} + 2a_{r-2} = f(r).$

where

$$f(r) = \begin{cases} 1 & \text{if } r = 5 \\ 0 & \text{otherwise.} \end{cases}$$ 2½+2½=5

(*b*) Find the solution of the recurrence relation :

$a_r = 4a_{r-1} - 3a_{r-2} + 2^r + r + 3$

with $a_0 = 1$, $a_1 = 4$. 4½