This question paper contains 4 printed pages]

Your Roll No. .....

5282

B.A. (Hons.) Programme J
DISCIPLINE CENTRED CONCURRENT COURSE
(Maths other than Economics)
(Algebra and Calculus)
(Admission of 2005 and onwards)

Time : 2 Hours **Maximum Marks : 38**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. **1** is compulsory and carries **8** marks. Attempt **six** more questions from the remaining Question Nos. **2** to **10,** selecting **two** questions each from Sections I, II and III. Each question carries **5** marks.

1. (a) Is the set $\left\{\begin{bmatrix}0\\-2\end{bmatrix}, \begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}2\\1\end{bmatrix}\right\}$ linearly dependent ? Justify.

   (b) Show that the join of the points (1, 2, 3), (4, 5, 7) is parallel to the join of the points (–4, 3, –6) and (2, 9, 2).

(c) Determine whether the function

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2}, & x \neq 2 \\ 3, & x = 2 \end{cases}$$

is continuous at $x = 2$.

(d) Find the length of the curve $y = x$ from $x = 1$ to $x = 4$. **(2 + 2 + 2 + 2)**

Section I

2. Test for consistency and solve

$$5x + 3y + 7z = 4$$

$$3x + 26y + 2z = 9$$

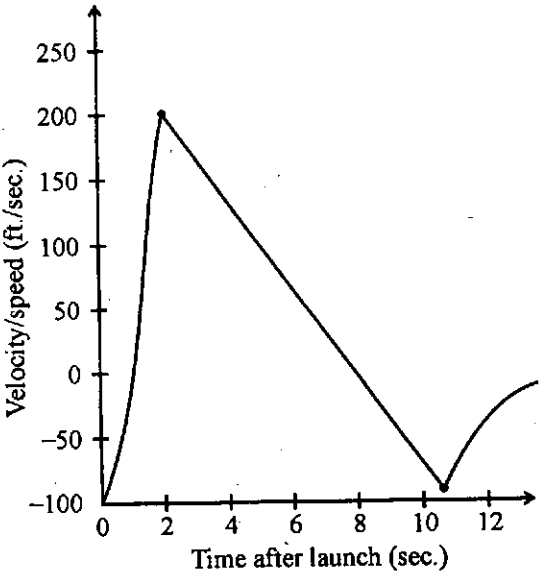
$$7x + 2y + 10z = 5$$ **5**

3. Find the vertex, focus and directrix of the parabola $x^2 - 6x - 6y + 6 = 0$. **5**

4. When a model rocket is launched, the propellant burns for a few seconds, accelerating the rocket upwards. After burnout, the rocket coasts upwards for a while and then begins to fall. A small explosive charge pops out of parachute shortly after the rocket starts down. The parachute slows the rocket to keep it from breaking when it lands. Use the graph below to answer the following. **5**

(i) How fast was the rocket climbing when the engine stopped ?

(ii) For how many seconds did the engine burn ?



Section II

5. Discuss concavity and convexity of the function $f(x) = \cos x$, on $[0, 2\pi]$. Also find its points of Inflexion. **5**

6. Verify the hypotheses and conclusion of Lagrange's Mean Value Theorem for the function $f(x) = \frac{x^3}{4} + 1$ on $[0, 2]$ **5**

7. Give MacLaurin's series for the function $f(x) = \sin 2x, x \in R$. **5**

Section III

8. Evaluate $\int \frac{1+\sqrt{x}}{1-\sqrt{x}}\,dx$ **5**

9. Show that if the length of the equatorial line on a Mercator projection is L, then the vertical distance D between the latitude lines at $\alpha°$ to $\beta°$ on the same side on the equator ($\alpha < \beta$) is **5**

$$D = \frac{L}{2\pi} \log \left| \frac{\sec \beta° + \tan \beta°}{\sec \alpha° + \tan \alpha°} \right|$$

10. Find a curve in the xy-plane through (0, 3) and whose tangent line at a point (x, y) has slope $\frac{2x}{y^2}$. **5**