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Your Roll No. ....

5288

B.A. (Hons.) Programme J
Discipline Centred Concurrent Course-
ECONOMICS
(For Economics Hons.)
(Maths : Linear Algebra and Calculus)
(Admission of 2005 and onwards)

Time : 2 Hours **Maximum Marks : 38**

(*Write your Roll No. on the top immediately on receipt of this question paper.*)

Attempt **six** questions in all, selecting **two** questions from each section.

SECTION – I

1. (a) Define vector space R^n over R. Show that :

   (i) If $\alpha \cdot x = 0$, then $\alpha = 0$ or $x = 0$ where $x \in R^n$, $\alpha \in R$.

   (ii) $(-\alpha)x = \alpha(-x) = -(\alpha x)$ where $a \in R$, $x \in R^n$. 4

   (b) Prove that the following subset S of R^3 is a spanning set of R^3, but not a basis of R^3

   $$S = \{(1, 0, 0), (1, 1, 0), (1, 1, 1), (0, 1, 0)\}$$ 4

2. (a) Is there a linear transformation

$T : R^3 \to R^2$ such that

$T(1, 0, 1) = (4, 6)$, $T(1, 1, 0) = (3, 5)$. If yes, find $T(1, 2, 3)$. 4

(b) Verify rank-nullity theorem for linear transformation

$T : R^3 \to R^3$ such that

$T(x, y, z) = (3x, x - y, 2x + y + z)$ 4

3. (a) Prove that $|\langle u, v\rangle| = \|u\| \, \|v\|$ iff u and v are linearly dependent. 4

(b) Define an orthonormal basis of R^3.

Prove that

$$\left\{\left(\frac{1}{3}, \frac{-2}{3}, \frac{-2}{3}\right), \left(\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}\right), \left(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}\right)\right\}$$

form an orthonormal set in R^3 w.r.t. standard inner product on R^n. 4

SECTION – II

4. Use $\in - \delta$ definition to prove that the following function is continuous at $x = 3$:

$$f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$$ 6

5. Let f be a continuous function on [a, b] and x_1, x_2, ..., x_n be points of [a, b]. Show that there exists a point $C \in [a, b]$ such that

$$f(C) = \frac{1}{n}\left[f(x_1) + f(x_2) + \ldots\ldots\ldots\ldots + f(x_n)\right]$$ **6**

6. Let function f be continuous on [a – b, a + b] and derivable in (a – b, a + b). Prove that there is a real number θ between 0 and 1 for which

$$f(a + b) - 2f(a) + f(a - b) = h[f'(a + \theta h) - f'(a - \theta h)]$$ **6**

SECTION – III

7. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be defined by setting :

$$f(x, y) = \begin{cases} \dfrac{xy}{\sqrt{x^2 + y^2}} & , \text{ if } (x,y) \neq (0, 0) \\ 0 & , \text{ if } (x,y) = (0, 0) \end{cases}$$

Show that f is continuous at (0, 0) but f is not differentiable at (0, 0). **5**

8. Find the maxima and minima of the function $4xy - x^4 - y^4$. **5**