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Your Roll No. ....

5287

B.A. (Hons.) Programme J

DISCIPLINE CENTRED CONCURRENT COURSE

ECONOMICS

(For Economics Hons.)

(Maths : Elements of Analysis)

(Admissions of 2005 and onwards)

Time : 2 Hours **Maximum Marks : 38**

(*Write your Roll No. on the top immediately on receipt of this question paper.*)

Attempt any **two** questions from each Section.

Section – I

1. (a) If

(i) $A = \{3, 3^2, 3^3, \dots 3^n, \dots\}$

(ii) $B = \left\{2, \frac{3}{2}, \frac{4}{3}, \dots \frac{n+1}{n}\right\}$

(iii) $C = \left\{\frac{(-1)^n}{n}, n \in N\right\}$

(iv) $D = \left\{1 + \frac{(-1)^n}{n}, n \in N\right\}$

Where N is the set of natural numbers. Find the supremum and infimum of the above sets, if they exist. 4

(b) Show that $\lim_{n \to \infty} \sqrt[n]{n} = 1$. 4

2. (a) Prove that every Cauchy's sequence is bounded and the converse need not be true. 4

(b) Show that the sequence $<a_n>$, where $a_n = 1 + \frac{1}{3} + \frac{1}{3^2} + \ldots + \frac{1}{3^{n-1}}$ converges. Find $\lim_{n\to\infty} a_n$ 4

3. (a) Define monotonic sequence. Show that the sequence $<a_n>$, where

$$a_n = \frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{n+n}$$

is convergent. 4

(b) State Cauchy's first theorem on limits. Apply it to show that

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \ldots + \frac{1}{\sqrt{n^2+n}}\right] = 1$$ 4

Section – II

4. (a) Test for convergence the series

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \sin\frac{1}{n^2}$$ 3

(b) Test for convergence the series whose n^{th} term is $\left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$ 3

5. Test the series $\frac{x}{2\sqrt{3}} + \frac{x^2}{3\sqrt{4}} + \frac{x^3}{4\sqrt{5}} + \ldots \infty$ for convergence for all values of x. 6

6. (a) State Leibnitz test for convergence of infinite series. Test for convergence and absolute convergence of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n\sqrt{n}} = 1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} + \dots \infty$$ 3

(b) Prove that the series $1 + r + r^2 + \dots (r > 0)$ converges if $r < 1$ and diverges if $r \geq 1$. 3

Section – III

7. Prove that if R is the radius of convergence of the power series $\Sigma a_n x^n$, then the series is absolutely convergent if $|x| < R$ and is divergent if $|x| > R$. 5

8. Find the radius of convergence and the exact interval of convergence of the following power series :

$$\Sigma \frac{(n!)^2 x^{2n}}{(2n)!}$$ 5

9. Show that

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, -1 \leq x \leq 1$$

and deduce that

$$\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$ 5