This question paper contains 3 printed pages.]

Your Roll No. ....

5286

B.A. (Hons.) Programme **J**

DISCIPLINE CENTRED CONCURRENT COURSE – ECONOMICS

(For Economics Hons.)

(Maths : Elements of Analysis)

(Admissions of 2005 and onwards)

Time : 2 Hours **Maximum Marks : 38**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any **two** questions from each section.

SECTION – I

1. (a) Define supremum and infimum of a set of real numbers. Find the supremum and infimum of the following sets if they exist : 4

(i) $\left\{1+\frac{(-1)^n}{n} : n \in N\right\}$ where N is the set of natural numbers.

(ii) $\left\{1, 1+\frac{1}{2}, 1+\frac{1}{2}+\frac{1}{2^2}, \ldots 1+\frac{1}{2}+\frac{1}{2^2}+\ldots+\frac{1}{2^{n-1}}\right\}$

(b) Show that the sequence $<a_n>$ defined by $a_n = r^n$ converges to zero if $|r| < 1$. 4

2. (a) If $<a_n>$, $<b_n>$ are sequences of real numbers such that

$\lim_{n\to\infty} a_n = a$, $\lim_{n\to\infty} b_n = b$, then prove that

$\lim_{n\to\infty} (a_n \cdot b_n) = a \cdot b$ **4**

(b) Prove that :

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

exists and lies between 2 and 3. **4**

3. (a) Show that every Cauchy sequence is bounded. Give an example of a bounded sequence which is not a Cauchy sequence. **4**

(b) Let $<a_n>$ be a sequence defined as follows :

$$a_1 = \frac{3}{2}, a_{n+1} = 2 - \frac{1}{a_n}, n \geq 1$$

Show that $<a_n>$ is monotonic and bounded. Find the limit of this sequence. **4**

SECTION – II

4. (a) Show that an infinite series Σu_n is convergent if and only if for each $\Sigma > 0$, there exists a positive integer m such that : **3**

$|u_{m+1} + u_{m+2} + ... + u_n| < \in$ for all $n \geq m$.

(b) Show that the series :

$$\sqrt{\frac{1}{4}} + \sqrt{\frac{2}{6}} + ... + \sqrt{\frac{n}{n+1}} + ...$$

does not converge. **3**

5. Test the series :

$$1+\frac{x^2}{2}+\frac{x^4}{4}+\frac{x^6}{6}+\ldots\ldots\ldots$$ **6**

for convergence, for all positive values of x.

6. (a) Show that every absolutely convergent series is convergent. Is the converge true ? **3**

(b) Test for convergence and absolute convergence the series :

$$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2}=\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots.$$ **3**

SECTION – III

7. Define power series and its radius of convergence. Determine the radius of convergence and exact interval of convergence of the following power series : **5**

$$\sum_{n=1}^{\infty}\frac{(x-1)^n}{2^n}$$

8. Show that :

$$\tan^{-1}x=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\ldots.$$

$-1\le x\le 1$ **5**

9. Define exponential function $E(x)$ as the sum of a power series. Show that the domain is the set of all real numbers. If e denotes $E(1)$, prove that $E(x)=e^x$, for all real x. **5**