Unit III (4)

Mechanics

5. (*a*) If the forces of magnitude P, Q and R act at a point parallel to the sides BC, CA and AB respectively of a triangle ABC, prove that the magnitude of the resultant is

$$[P^2 + Q^2 + R^2 - 2QR \cos A - 2\, PR \cos B - 2PQ \cos C]^{1/2}.$$ 6

(*b*) Find the centre of gravity of the area enclosed by the curves $y^2 = ax$ and $x^2 = by$. 6

Or

(*a*) Three forces P, Q, R act along the sides BC, AC and BA of an equilateral triangle ABC. If their resultant is a force parallel to BC through the centroid of the triangle, prove that :

$$Q = R = \frac{1}{2} P. \qquad 6$$

(b) A uniform beam of length '$2a$' rests in equilibrium against a smooth vertical wall and upon a smooth peg at a distance 'b' from the wall. Show, by the principle of virtual work that in the position of equilibrium the beam is inclined to the wall at an angle

$$\sin^{-1} (b/a)^{1/3}. \qquad 6$$

6. (a) Two cars start off to race with velocities v_1 and v_2 and travel in a straight line with uniform

accelerations f_1 and f_2. If the race ends in a dead heat, prove that the length of the course is

$$\frac{2(v_1 - v_2)(v_1 f_2 - v_2 f_1)}{(f_1 - f_2)^2}.$$ 6

(b) A particle is projected from the lowest point of a smooth vertical circle along its inside with a velocity

$$\sqrt{\frac{7gr}{2}},$$

where r is the radius of the circle. Show that the particle leaves the circle after reaching a height $\frac{3}{2}r$ and then returns to the point of projection. Find also the latus rectum of the parabola subsequently described by the particle. 6

Or

(*a*) If a particle moves along a straight line according to the law

$$v^2 = ax - bx^3,$$

prove that :

$$27bv^4 = 4(a - 2f)(a + f)^2,$$

where 'a' and 'b' are constant and f is the acceleration. 6

(*b*) A particle describes an elliptic orbit under a central force towards one focus S. If v_1 is the speed at the end B of the minor axis and v_2, v_3 the speeds at the ends A, A′ of the major axis, show that :

$$v_1^2 = v_2 v_3.$$ 6

Unit III (5)

Theory of Games

5. (*a*) Solve graphically the following linear programming problem : 6

Maximize : $Z = 6x_1 + x_2$

subject to :

$$2x_1 + x_2 \geq 3$$

$$x_1 - x_2 \geq 0$$

$$x_1, x_2 \geq 0$$

(*b*) Use Charne's Big M method to solve the following linear programming problem : 6

Maximize : $Z = 6x_1 + 4x_2$

subject to :

$$2x_1 + 3x_2 \leq 30$$

$$3x_1 + 2x_2 \leq 24$$

$$x_1 + x_2 \geq 3$$

$$x_1, x_2 \geq 0$$

Or

(*a*) Solve by Simplex method the problem : 6

Maximize : $Z = 5x_1 + 3x_2$

subject to :

$$x_1 + x_2 \leq 2$$

$$5x_1 + 2x_2 \leq 10$$

$$3x_1 + 8x_2 \leq 12$$

$$x_1, x_2 \geq 0$$

(*b*) Write down the dual of the following problem in a form so that the dual variables are non-negative : 6

Maximize : $Z = 5x_1 + 12x_2 + x_3$

subject to :

$$x_1 + 2x_2 + x_3 \leq 5$$

$$2x_1 - x_2 + 3x_3 = 3$$

$x_1, x_2 \geq 0$, x_3 is unrestricted.

6. (*a*) Consider the game G with the pay-off matrix :

$$\begin{bmatrix} 2 & 6 \\ -2 & \lambda \end{bmatrix}.$$

show that G is determinable whatever λ may be. Determine the value of G. 6

(*b*) Use graphical method to solve the rectangular game whose pay-off matrix is : 6

$$\begin{bmatrix} -2 & 0 \\ 3 & -1 \\ -3 & 2 \\ 5 & -4 \end{bmatrix}.$$

Or

(*a*) Solve the game whose pay-off matrix is given by : 6

$$\begin{bmatrix} 1 & 3 & 6 \\ 2 & 1 & 3 \\ 6 & 2 & 1 \end{bmatrix}.$$