Sr. No. of QP	:623 E
Unique Paper Code	: 235685
Name of the Paper	: Elements of Analysis
Name of Course	: B.A. (H) [redacted]
Semester	: VI
Duration	: 3 Hours
Maximum Marks	: 75 Marks

Instructions for Candidates

1. In all there are six Questions.
2. In Question No. 1 to 5 attempt any two parts. Each part carries 6 marks.
3. In Question No. 6 attempt any five parts. Each part carries 3 marks.

1. (a) Prove that for any two real numbers a and b
$$\big|\,|a| - |b|\,\big| \le |a - b|$$
Show that equality holds if and only if a, b are of same sign.

(b) Define a bounded set. Give an example of
(i) a countable bounded set.
(ii) an uncountable bounded set.
(iii) a countable unbounded set.

(c) Define a convergent sequence ,Show that a convergent sequence converges to a unique limit.

2. (a) Define cluster points of a sequence. Determine cluster points of the sequences
(i) $\langle (-1)^n \left(1+\frac{1}{n}\right)\rangle$ (ii) $\langle \frac{\cos n\pi}{2} \rangle$

(b) Show that the sequence $\langle (1+\frac{1}{n})^n \rangle$ is convergent.

(c) If $\langle S_n \rangle$ is the sequence defined by
$S_1 = 0, S_2 = 1,\ S_{n+2} = \frac{1}{2}(S_n + S_{n+1}),$
Show that $\langle S_n \rangle$ is convergent, also find $\lim_{n\to\infty} S_n$.

3. (a) Define sequence of partial sums of an infinite series $\sum_{n=1}^{\infty} a_n$. For the series
$$\frac{1}{a(a+b)} + \frac{1}{(a+b)(a+2b)} + \frac{1}{(a+2b)(a+3b)} + \dots\dots\dots\dots\dots\dots\dots$$
find its sequence of partial sums. What is the sum of the series?

(b) State comparison test for convergence of a positive term series. Show that the series $\sum_{n=1}^{\infty} \frac{|\cos nx|}{n^2}$ is convergent for any real number x.

(c) State ratio test for convergence of a positive term series. Discuss convergence of the series $\sum_{n=1}^{\infty} \frac{x^n}{n^p}$ where $x > 0,\ p > 0$

4. (a) Is the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{\sqrt{n}}$ an alternating term series? After stating Leibnitz's test show that the above series is convergent.

(b) What is an absolutely convergent series? Show that the geometric series $\sum_{n=0}^{\infty} r^n$ is absolutely convergent for $|r| < 1$. Is it conditionally convergent for some value of r ?

(c) Discuss the convergence of the series $\sum_{n=1}^{\infty} a_n$, where
(i) $a_n = \sqrt{n^4+1} - n^2$ (ii) $a_n = (-1)^n \frac{1}{n^p}$ $(p > 0)$

5. (a) Define a power series. Which of the following are power series ? If yes , find its radius of convergence
(i) $\sum_{n=0}^{\infty} \frac{x^n}{n+1}$ (ii) $\sum_{n=1}^{\infty} \frac{x^{1/n}}{n+1}$ (iii) $\sum_{n=0}^{\infty} \frac{x^{n!}}{n}$

(b) What do you mean by term-by-term differentiation of power series? Show that radius of convergence is unaffected under term-by-term differentiation.

(c) Starting from power series expansion of exp (x), prove that
(i) exp(x + y) = exp (x) exp (y)
(ii) exp (x) > 0 for all real x
(iii) $\frac{d}{dx}$ (exp (x)) = exp (x)

6. (i) Write True or False , justifying your answer
$|x| = \sqrt{x^2} = \max\{x, -x\}$

(ii) Is the sequence $\langle 0,1,0,2,0,3, \ldots\ldots \rangle$ convergent? Justify.

(iii) If $a > 0$, and $\langle a_n \rangle$ is defined as $a_1 = \sqrt{a},\ a_{n+1} = \sqrt{a_n \cdot a}$,
show that $\lim_{n\to\infty} a_n = a$

(iv) Give an example of a divergent series $\sum_{n=1}^{\infty} a_n$ for which $\lim_{n\to\infty} a_n = 0$

(v) What is the sum of the series
$$1 + \frac{1}{3} + \frac{1}{4} + \frac{1}{27} + \frac{1}{16} + \frac{1}{243} + \frac{1}{64} + \ldots\ldots\ldots ?$$

(vi) Using power series expansion , prove that $\cos^2 x + \sin^2 x = 1$.

(vii) Starting from sum of a geometric series, obtain a power series expansion of $\log(1+x)$; $-1 < x < 1$.