This question paper contains 4+1 printed pages]

Your Roll No. ....

6732

B.A./B.Sc. (Hons.)/I **D**

MATHEMATICS—Unit II

(Algebra–I)

(Admissions of 2008 and before)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each Section.

All questions carry equal marks.

SECTION I

1. (*a*) Prove that the equation :

$$a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ag \cos \theta + 2bf \sin \theta + c = 0$$

has four roots and the sum of these values of θ is an even multiple of π.

P.T.O.

(*b*) Sum to n terms of the series :

$$\sin\theta\sin\theta + \sin^2\theta\sin 2\theta + ... + \sin^n\theta\sin n\theta .$$

(*c*) If n is a positive integer, prove that :

$$(1+\sqrt{3})^n + (1-\sqrt{3})^n = 2^{n+1}\cos(n\pi/3)$$

and hence find the value when $n = 9$.

SECTION II

2\. (*a*) (*i*) Prove that every Hermitian matrix can be expressed as P + *i*Q, where P and Q are real symmetric and real skew symmetric matrices respectively.

(*ii*) Prove that every skew symmetric matrix of odd order is singular.

(*b*) Reduce the matrix :

$$A = \begin{pmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{pmatrix}$$

to the normal form $\begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}$ and hence determine its rank.

(*c*) Verify that the matrix :

$$A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

satisfies its own characteristic equation. Is it true for every square matrix. Hence or otherwise obtain A^{-1}.

SECTION III

3. (*a*) Solve completely the system of equations :

$$x + 3y + 4z + 7w = 0$$

$$2x + 4y + 5z + 8w = 0$$

$$3x + y + 2z + 3w = 0.$$

(*b*) If α, β, γ, δ be the roots of the equation :

$$x^4 + px^3 + qx^2 + rx + s = 0.$$

then find the value of :

(*i*) $\sum\alpha^2\beta$

(*ii*) $\sum\alpha^2\beta\gamma$

(*iii*) $\sum\alpha^2\beta^2$.

(*c*) (*i*) Solve the equation :

$$x^3 - 6x^2 + 11x - 6 = 0$$

the roots being in arithmetic progression.

(*ii*) Solve the equation :

$$x^4 - 12x^3 + 49x^2 - 78x + 40 = 0$$

by removing its second term.

SECTION IV

4. (*a*) If g.c.d $(a, b) = 1$, show that :

$$\text{g.c.d}(a^n, b^n) = 1 \text{ and } \text{g.c.d}(a+b, a-b) = 1 \text{ or } 2.$$

(*b*) (*i*) Compute $a^{-1}ba$ where :

$a = (135)$ and $b = (1579)$.

(*ii*) Determine which of the following permutation are even :

$f = (1\ 2\ 3\ 4\ 5)\ (1\ 2\ 3)\ (4\ 5)$

and $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 5 & 4 & 3 & 6 & 1 & 7 & 9 & 8 \end{pmatrix}$.

(*c*) (*i*) Show that every permutation on a finite set can be expressed as a product of disjoint cycles.

(*ii*) Given that :

$f = (1\ 2\ 3\ 4)\ (2\ 3\ 5)\ (1\ 3\ 4)$

express f as a product of disjoint cycles and also write f as a product of transpositions.