

This question paper contains 4+1 printed pages]

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S. No. of Question Paper : 8146

Unique Paper Code : 235683

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Name of the Paper : Mathematics I (Algebra & Calculus) other than Economics

Name of the Course : B.A. (Hons.) III, (Discipline Centred Concurrent Course)

Semester : VI

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. 1 is compulsory and carries 15 marks.

Attempt six more questions selecting

any two questions from each section.

Each question carries 10 marks.

1. (i) Find the matrix A such that :

$$A \begin{bmatrix} 3 & 4 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 8 \\ 9 & 4 \end{bmatrix}$$

(ii) Let

$$f(x) = \frac{x}{|x|}.$$

Find  $\lim_{x \rightarrow 0} f(x)$  if it exists.

P.T.O.

(iii) Find

$$\frac{dy}{dx},$$

when

$$x^3 + y^3 = 3xy.$$

(iv) Show that :

$$\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2 (3y+k)$$

(v) Evaluate :

$$\int x^3 e^{x^4} dx.$$

5×3

### SECTION I

2. (i) Show that the matrix  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  satisfies the equation  $A^2 - 5A - 2I = 0$ . Hence obtain  $A^{-1}$ . 5

(ii) Solve the following system of linear equations using Cramer's Rule :

$$5x - 7y + z = 11$$

$$6x - 8y - z = 15$$

$$3x + 2y - 6z = 7$$

5

3. (i) Find the co-ordinates of the foci, the vertices, the lengths of major and minor axes and the eccentricity of the ellipse  $36x^2 + 4y^2 = 144$ . Also sketch the graph of the ellipse. 5

- (ii) If

$$A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}, B = [4 \ 6],$$

verify that  $(AB)' = B'A'$ . Where  $A'$  and  $B'$  are transpose of matrix  $A$  and  $B$  respectively. 5

4. (i) Find the centre and radius of the following circle :

$$x^2 + y^2 - 4x - 8y - 45 = 0 \quad 5$$

- (ii) Find the coordinates of the focus, axis of the parabola, the equation of the directrix and the length of latus rectum of the parabola  $y^2 = 10x$ . Also the sketch the graph of the parabola. 5

### SECTION II

5. (i) If

$$y = (\sin x)^{\cos x},$$

find

$$\frac{dy}{dx}$$

5

- (ii) Examine the continuity at  $x = 1$  and  $x = 2$  of the function : 5

$$f(x) = \begin{cases} 2x & , 0 \leq x \leq 1 \\ 2 - x & , 1 < x \leq 2 \\ x^2 - 2x & , x > 2 \end{cases}$$

6. (i) Examine the function  $f(x) = x^3 - 3x^2 + 3x - 3$  for concavity and points of inflection. 5

- (ii) Show that the function :

$$f(x) = x^2 + \frac{250}{x}$$

has a minimum value at  $x = 5$ . 5

7. (i) Verify Rolle's theorem for : 5

$$f(x) = x^3 - 4x, \text{ in } [-2, 2]$$

- (ii) Write down the Maclaurin series expansion for the function  $f(x) = e^x$ . 5

### SECTION III

8. (i) Find the area of the region bounded by the curves  $y = x^2$  and  $y = x$ . 5

- (ii) Find : 5

$$\int \frac{x}{(x+1)(x-2)} dx.$$

9. (i) Find the general solution of the differential equation :

5

$$\frac{dy}{dx} = \frac{1 + y^2}{1 + x^2}.$$

- (ii) The marginal cost function of a firm is  $MC = 3 + 2 \log x$ . Find the total cost function, when the cost of producing 1 unit is Rs. 21.

5

10. (i) Evaluate :

5

$$\int_0^1 \frac{\tan^{-1} x}{1 + x^2} dx.$$

- (ii) Show that the series  $\sum_{n=1}^{\infty} 4^n$  is divergent.

5