This question paper contains **8** printed pages]

Your Roll No. .....

6749

B.A. (Hons.)/B.Sc. (Hons.)/III **D**

MATHEMATICS—Paper XVII and XVIII (*iii*)

(Discrete Mathematics)

Time : 2 *Hours* *Maximum Marks* : 38

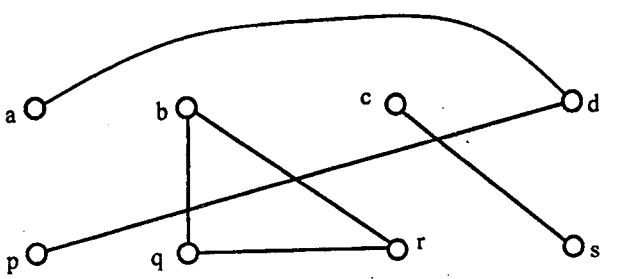
(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions.

Section 1

1. Attempt any *two* parts : 3+3

(*a*) Define a subgraph and a connected component of a graph. For the given graph G, find the connected components.



P.T.O.

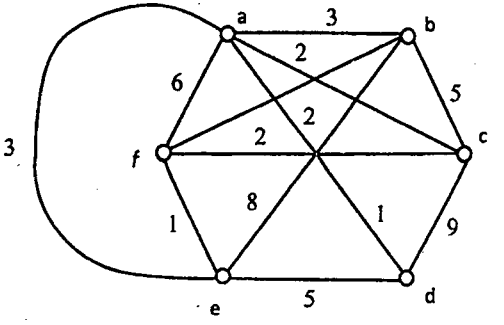
(*b*) Show that for any graph, sum of degrees of all the vertices is equal to the number of edges. Deduce that if graph G be K-regular graph with n vertices and m edges, then $2m = nK$.

(*c*) In a connected planar graph with 'e' edges and 'v' vertices, show that :

$$3v - e \geq 6$$

Hence deduce that K_5 is not planar.

2. Apply nearest neighbourhood method to find a Hamiltonian circuit of minimum length in the following graph starting at the vertex 'a'. 3½



Section 2

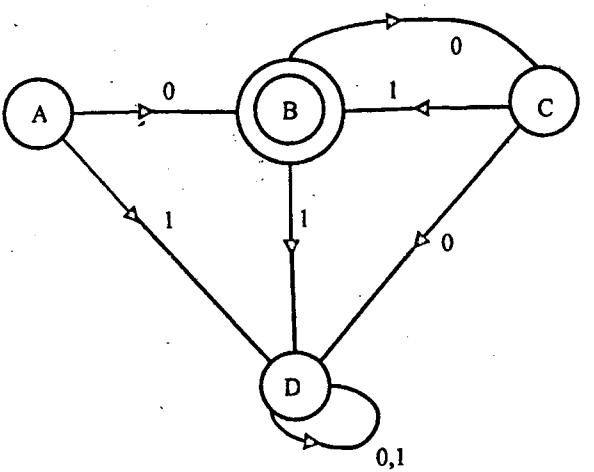
3. Attempt any *two* parts : 3+3

(*a*) For the finite state machine (FSM) in the figure given below, do the following :

(*i*) Identify the accepting and the rejecting states.

(*ii*) Operate the machine on the input sequence 0 1 1 1 0 1 1 0 1 and write the corresponding state sequence and the output sequence.

(*iii*) Give its tabular representation.



(*b*) For the following FSM, find all 0 equivalent and 1 equivalent states.

State	Input		Output
	0	2	
A	B	C	0
B	F	D	0
C	G	E	0
D	H	B	0
E	B	F	1
F	D	H	0
G	E	B	0
H	*B*	C	1

(*c*) Design a FSM that accepts all binary sequences ending with the digits 0 1 1.

4. Design a modulo 3 counter that receives a sequence of 0's, 1's and 2's as input and produces a sequence of 0's, 1's and 2's as output such that at any instant, the output is equal to the modulo 3 sum of the digits in the input sequence. 3½

Section 3

5. Attempt any *two* parts : 3+3

(*a*) If

$$a = (a_r),\ b = (b_r) \text{ and } c = (c_r)$$

by any three numeric functions such that :

$$c = ab,$$

show that :

$$(\nabla c)_r = (\nabla a)_r b_r + a_{r-1}(\nabla b)_r,\ r \geq 1$$

Hence determine

$\nabla(ab)$, if $a_r = r + 1$ and $b_r = \alpha^r$, $r \geq 0$.

(*b*) If

$$A = [a_{ij}]$$

be the incidence matrix of a (b, v, r, k, λ) configuration, then prove that :

(*i*) $a_{ij}^2 = a_{ij}$, $\forall i$ and j

(*ii*) $\sum_{i=1}^{b} a_{ij} = r$, $\forall j$ and

(*iii*) $\sum_{j=1}^{v} a_{ij} = k$, $\forall i$.

(*c*) Let :

$$B = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

be a normalized Hadamard matrix. Derive from B, an 8×8 Hadamard matrix and hence derive a (7, 3, 1) configuration.

6. From the following binary sequences, generate a code system of 16 words which detects upto three errors and corrects

upto one error : 3½

0	1	1	0	1	0	0
0	0	0	1	1	0	1
1	1	0	1	0	0	0
1	0	1	0	0	0	1
0	0	1	1	0	1	0
1	0	0	0	1	1	0
0	1	0	0	0	1	1

Section 4

7. Attempt any *two* parts : 3+3

(*a*) Determine the discrete numeric function corresponding to the generating function :

$$A(z) = \frac{1}{5 - 6z + z^2}.$$

(*b*) Find a simple expression for the generating function of the discrete numeric function :

$$1, 2/3, 3/9, 4/27, \ldots\ldots, (r+1)/3^r.$$

(*c*) Find the particular solution of the difference equation :

$$a_r + 5a_{r-1} + 6a_{r-2} = 3r^2 - 2r + 1.$$

8. Solve the difference equation :

$$a_r - 5a_{r-1} + 6a_{r-2} = 2^r + r,\ r \geq 0$$

by the method of generating function. 3½