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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8149**

Unique Paper Code : **235686** **D**

Name of the Paper : **Mathematics IV (Linear Algebra and Calculus)**

Name of the Course : **B.A. (Hons.) – III-(For Economics Hons.)**

Semester : **VI**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *six* questions in all, selecting *two* questions from each Section.

Section I

1\. (*a*) Let $V = \mathbf{R}^2$, for $(a_1, a_2), (b_1, b_2) \in \mathbf{R}^2$, define addition and scalar multiplication on $\mathbf{R}^2$ as under :

$$(a_1, a_2) + (b_1, b_2) = (0, a_2 + b_2)$$

$$\alpha(a_1, a_2) = (\alpha a_1, \alpha a_2) \ \forall \ \alpha \in \mathbf{R}$$

Show that $\mathbf{R}^2$ is not a vector space over $\mathbf{R}$ under the above defined operations. 6

(*b*) If $v_1 = (1, 2, 1)$, $v_2 = (3, 1, 5)$, $v_3 = (3, -4, 7)$ are vectors in $\mathbf{R}^3$, prove that :

$$\text{Span } \{v_1, v_2\} = \text{Span } \{v_1, v_2, v_3\}.$$ 6

(*c*) Find a basis and dimension of the subspace :

$$W = \{(a, b, c) : 2a + b - c = 0\} \text{ of } \mathbf{R}^3.$$ 4

P.T.O.

2. (*a*) Does the function $T : \mathbf{R}^3 \to \mathbf{R}^3$ defined by :

$$T(x, y, z) = (x - y, x^2, 2z)$$

a linear transformation ? Justify your answer. 4

(*b*) Find a linear transformation $T : \mathbf{R}^2 \to \mathbf{R}^2$ such that :

$$T(2, -1) = (1, 1), T(-1, 1) = (2, 3).$$

Find T(9, 2) as well. 6

(*c*) $T : \mathbf{R}^3 \to \mathbf{R}^3$ is a linear operator, the matrix A of which with respect to standard basis is :

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{pmatrix}.$$

Find a basis for the null space of T and its nullity. 6

3. (*a*) Let $u = (\alpha_1, \alpha_2)$, $v = (\beta_1 \beta_2) \in \mathbf{R}^2$, define

$$\langle u, v \rangle = \alpha_1 \beta_1 + 3\alpha_2 \beta_2$$

Does it define an inner product on $\mathbf{R}^2$? Explain. 6

(*b*) State Cauchy-Schwarz inequality. Verify the same for the vectors :

$$U = (1, -2, 0, 2), V = (-3, 6, 0, 6).$$

Does the equality occur ? If so, why ? 6

(*c*) If A is unitary, prove that A^{-1} is unitary. 4

Section II

4. (*a*) Use $\in - \delta$ definition to show that :

$$\lim_{x \to 0} x \sin \frac{1}{x}$$

exist and equals 0. 6

(*b*) Determine the points of discontinuity of :

$$f(x) = x[x], \quad \forall\ x \in \mathbf{R}.$$ 6

5. (*a*) Let :

$$f(x) = |x| + |x - 1|, \quad \forall\ x \in \mathbf{R}.$$

Show that f is derivable at all points, except $x = 0, 1$. 6

(*b*) Discuss the differentiability of the function :

$$f(x) = \begin{cases} x & x < 1 \\ 2 - x & 1 \le x \le 2 \\ -2 + 3x - x^2 & x > 2 \end{cases}$$

at $x = 1, 2$. 6

6. (*a*) State Rolle's Theorem and verify the same for the function :

$$f(x) = x^3 - 6x^2 + 11x - 6, \text{ in } [1, 3].$$ 6

(*b*) Verify Lagrange's Mean Value Theorem for the function :

$$f(x) = x(x - 1)(x - 2), \text{ in } \left[0, \frac{1}{2}\right].$$ 6

Section III

7. Let f be defined as :

$$f(x, y) = \begin{cases} \dfrac{x^2 - y^2}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

(a) Show that both :

$$\lim_{x \to 0} \lim_{y \to 0} f(x, y) \text{ and } \lim_{y \to 0} \lim_{x \to 0} f(x, y)$$

exist, but are unequal.

(b) Is $f(x, y)$ continuous at origin ? Justify. 9½

8. Let :

$$f(x, y) = \begin{cases} \dfrac{xy(x^2 - y^2)}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

Prove or disprove $f_{xy}(0, 0) = f_{yx}(0, 0)$. 9½

9. (a) Use Taylor's Theorem to expand $x^4 + x^2y^2 - y^4$ about the point (1, 1) upto the terms of second degree. 5½

(b) Find all critical points of the function :

$$f(x, y) = (x - 1)^2 - y^2$$

Also classify them as maxima/minima/saddle point(s). 4