This question paper contains **4** printed pages]

Roll No. ☐☐☐☐☐☐☐☐

S. No. of Question Paper : **5048**

Unique Paper Code : **237662**   **D**

Name of the Paper : **Sample Surveys and Design of Experiments (STP-606)**

Name of the Course : **B.Sc. (Mathematical Sciences)**

Semester : **VI**

Duration : **3 Hours**   Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *Five* questions in all, selecting *two* from

Section A and B each and *one* question from Section C.

Section A

1. (*a*) Discuss the basic principles of sample surveys. What are the main steps involved in a sample survey ? Discuss them briefly.

    (*b*) Prove that, in srswor, the variance of the sample mean is given by :

$$\text{Var}(\bar{y}_n) = \left(\frac{1}{n} - \frac{1}{N}\right)S^2,$$

The notations have their usual meaning.

Prove that srswor provides a more efficient estimator of population mean relative to srswr.   6,9

P.T.O.

2. (*a*) In stratified random sampling with given cost function of the form :

$$C = a + \sum_{i=1}^{k} C_i n_i,$$

where '*a*' is the overhead cost and C_i is the cost per unit in the *i*th stratum, prove that $\text{Var}(\bar{y}_{st})$ is minimum if :

$$n_i = \frac{N_i S_i}{\sqrt{C_i}}$$

Also, discuss the case when $C_i = C_o \ \forall \ i = 1, 2 \ldots\ldots, k.$

(*b*) Define difference estimator and derive from it the regression estimator. Show that for a super-population model, the regression estimator is BLUE.   7,8

3. (*a*) Prove that, in the presence of linear trend, the variance of a stratified sample is only $\frac{1}{n}$ th of the variance of a systematic sample and the latter is also approximately $\frac{1}{n}$ th the variance of a random sample. Hence show that the systematic sample with Yates' End Correction provides the exact population mean.

(*b*) Define proportional allocation and obtain the variance of estimated population mean under proportional allocation. Also compare it with the variance of estimated population mean under srswor.   9,6

Section B

4. (a) Explain the following terms which occur in Design of Experiments :

(i)   Experimental errors

(ii) Uniformity trials and

(iii) Shape and size of blocks and plots.

(b) Estimate a missing value in a RBD. Calculate the standard error of the difference between two treatment means, one of which involves the missing plot.   6,9

5. (a) Describe the technique of analysis of variance. Work out the analysis of variance for two-way classification with one observation per cell.

(b) What is LSD ? Discuss its advantages and disadvantages. Determine the efficiency of this design relative to RBD with rows as blocks.   6,9

6. (a) Present the analysis of a 2^3 factorial experiment conducted in a RBD with r replications.

(b) How is the efficiency of a design measured ? Determine the efficiency of RBD over CRD.   8,7

Section C

7. (a) Obtain the least squares estimates of β_0 and β_1 in the model :

$$y = \beta_0 + \beta_1 x + \varepsilon$$

stating the underlying assumptions. Prove that these estimates are unbiased and have minimum variance.

(b) Write a note on NSSO of India.   10,5

P.T.O.

8. (a) For the simple linear regression model :

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i (i = 1, 2, \ldots\ldots, n),$$

where $\varepsilon_i \sim NID (0, \sigma^2)$:

(i) Test the significance of the regression in the model.

(ii) Show that :

$$\text{Cov}(\beta_0, \beta_1) = - (\bar{x}\sigma^2 / S_{xx}),$$

notations have their usual meaning.

(b) Distinguish between de facto and de jure methods of population count in India. Also write a note on population statistics in India. 10,5