[This question paper contains 4 printed pages.]

4488-A *Your Roll No.* ....

B.A./I **AS**

(L)

MATHEMATICS – Paper I

(Algebra and Calculus)

(Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks : 100*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :– The maximum marks printed on the question paper are applicable for the students of Category 'B'. These marks will, however, be scaled down proportionately in respect of the students of Category 'A' at the time of posting of awards for compilation of result.

All *Sections are compulsory and have equal marks. Attempt any* **two** *parts from each Section.*

PART A (Algebra)

SECTION I

1. (a) If G is a group such that every element of the group G is its own inverse, then show that G is abelian.

P.T.O.

(b) Show that the intersection of two normal subgroups of a group G is a normal subgroup of G.

(c) Let R be a ring with unity. If

$$(a\,b)^2 = a^2 b^2, \quad \forall\ a, b \in R,$$

then show that R is a commutative ring.

SECTION II

2. (a) Define linearly dependent set of vectors over a field F. Prove that the following subset of $R^3(R)$ is linearly independent.

$\{(1, -1, 1), (4, 1, 0), (8, 1, 1)\}$.

(b) Find the rank of the matrix :

$$\begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$$

(c) Obtain the characteristic equation of the matrix

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$$

and use Cayley-Hamilton theorem to find its inverse.

SECTION III

3. (a) The roots of the equation

$$3x^3 - x^2 - 3x + 1 = 0,$$

are in H.P. Find them.

(b) If, α, β, γ be the roots of the equation :

$x^3 + qx + r = 0$, such that no two of them are equal in magnitude but opposite in sign, find the values of :

(i) $\sum \frac{1}{\beta+\gamma}$ (ii) $\sum \frac{\beta^2+\gamma^2}{\beta+\gamma}$

(c) If $Z = \cos\theta + i\sin\theta$, show that

$$\frac{Z^{2n}-1}{Z^{2n}+1} = i\tan n\theta,$$

n being an integer.

PART B (Calculus)

SECTION IV

4. (a) Examine the continuity of the function f defined as

$$f(x) = \begin{cases} 2 - x & , \text{ if } x \le 2 \\ -2 + 3x - x^2, & \text{if } x > 2 \end{cases}$$

at $x = 2$.

(b) If $y = \tan^{-1}x$, show that

$$(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + n(n + 1)y_n = 0$$

(c) If $u = \log \frac{x^2 + y^2}{x + y}$, then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1.$$

SECTION V

5. (a) Prove that the sum of the intercepts on the coordinate axes of any tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is constant.

(b) Find all the asymptotes of the curve

$$y^3 + x^2y + 2xy^2 - y + 1 = 0.$$

(c) Trace the curve

$$y^2(a + x) = x^2(3a - x)$$

SECTION VI

6. (a) Obtain a reduction formula for

$$\int x^m \sin nx \, dx, \quad m, n \in \mathbb{N}$$

(b) Find the circumference of the circle

$$x^2 + y^2 = r^2.$$

(c) Find the surface area formed by the revolution of the loop of the curve $3ay^2 = x(x - a)^2$ about x-axis.