This question paper contains 4+2 printed pages]

Your Roll No...............

4488

B.A./I AS

(L)

MATHEMATICS—Paper I

(Algebra and Calculus)

Time : 3 Hours *Maximum Marks : 100*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :— The maximum marks printed on the question paper are applicable for the students of the (Cat. 'B'). These marks will, however, be scaled down proportionately in respect of the students of Regular Colleges at the time of posting of awards for compilation of result.

All questions are compulsory and have equal marks.

Attempt any *two* parts from each question.

1. (*a*) Define linear independence of a set of vectors in a vector space over a field. Show that if x, y, z are linearly independent vectors of a vector space over the field **C**

P.T.O.

of complex numbers, then $x+y, y+z, z+x$ are also linearly independent over **C**.

(*b*) Find the rank of the matrix

$$\begin{pmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{pmatrix}.$$

(*c*) Find the characteristic equation of the matrix

$$A = \begin{pmatrix} 1 & -1 & 1 \\ 4 & 1 & 0 \\ 8 & 1 & 1 \end{pmatrix}.$$

Hence find A^{-1}.

2\. (*a*) Prove that :

$$\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta.$$

(*b*) Solve the equation :

$$z^4 + 4z^2 + 16 = 0 \text{ in } \mathbf{C}.$$

(*c*) If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the values of

(*i*) $\Sigma\alpha^2\beta^2$

(*ii*) $\sum\frac{\alpha}{\beta}$

(*iii*) $\Sigma\alpha^2\beta$.

3. (*a*) Is the function f where

$$f(x) = (x - a)\sin\frac{1}{x - a} \text{ for } x \neq a,$$

$$f(a) = 0$$

continuous and derivable at $x = a$?

(*b*) If

$$u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}; \; x^2 + y^2 + z^2 \neq 0$$

show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0.$$

(c) If

$$x = 2 \cos t - \cos 2t \text{ and}$$

$$y = 2 \sin t - \sin 2t,$$

find the value of $\frac{d^2 y}{dx^2}$ when $t = \frac{\pi}{2}$.

4. (a) Find the condition for the curves

$$ax^2 + by^2 = 1,$$

$$a_1 x^2 + b_1 y^2 = 1$$

to intersect orthogonally.

(b) Find the position and nature of the double points on the following curve :

$$y(y - 6) = x^2(x - 2)^3 - 9.$$

(*c*) Trace the curve

$$y^2(a^2 + x^2) = x^2(a^2 - x^2).$$

5. (*a*) Explain why Lagrange's Mean Value Theorem is not applicable to the function

$$f(x) = 2 - (x - 2)^{2/3},\ x \in [0, 3].$$

(*b*) If

$$x_1, x_2, \ldots\ldots, x_n$$

are given *n* real numbers, find '*a*' so that :

$$\sum_{i=1}^{n} (x_i - a)^2$$

is minimum.

(*c*) Evaluate :

(*i*) $\lim_{x \to 0} x^x$;

(*ii*) $\lim_{n \to 0} \dfrac{x - \dfrac{x^3}{6} - \sin x}{1 - \cos x}$

6. (*a*) Evaluate :

(*i*) $$\int \frac{x\,dx}{(x+2)\sqrt{x+1}};$$

(*ii*) $$\int_0^{\infty} \frac{x^3 dx}{(1+x^2)^{n/2}} \quad (n > 4).$$

(*b*) Find the area of the loop of the folium $x^3 + y^3 = 3axy$.

(*c*) Find the volume of the solid generated by rotating the arch of the parabola $y = x^2$, $0 \le x \le 2$ about X-axis.