[This question paper contains 4 printed pages.]

4489-A *Your Roll No.* .....

B.A. / I **AS**

(R)

MATHEMATICS – Paper I

(Algebra and Calculus)

(Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks : 100*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :– The maximum marks printed on the question paper are applicable for the students of Category 'B'. These marks will, however, be scaled down proportionately in respect of the students of Regular Colleges, Category 'A' at the time of posting of awards for compilation of result.

All *Sections are compulsory and have equal marks. Attempt any* **two** *parts from each Section.*

PART A (Algebra)

SECTION I

1. (a) Prove that if G is an abelian group, then

$$(a\,b)^n = a^n\,b^n$$

for all $a, b \in G$ and for every positive integer n.

P.T.O.

(b) Define a group and a subgroup of a group. Show that a non-empty subset H of a group G is a subgroup if and only if $ab^{-1} \in H \ \forall \ a, b \in H$.

(c) In a ring $(R, +, \cdot)$, show that

(i) $a \cdot 0 = 0 \cdot a = a$

(ii) $a(-b) = (-a) = -ab$

SECTION II

2. (a) Show that the set

$$S = \left\{ \begin{pmatrix} a & 0 \\ a & 0 \end{pmatrix} : a \in \mathbb{R} \right\}$$

is a subspace of the vector space of 2×2 real matrices.

(b) Find the characteristic equation of the matrix

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{pmatrix}$$

and find its inverse.

(c) Find the rank of the matrix

$$\begin{pmatrix} 1 & -1 & 3 \\ 2 & 2 & 4 \\ 3 & 1 & 7 \end{pmatrix}$$

SECTION III

3. (a) Prove that

$$\sin 6\theta = 6\cos^5\theta\sin\theta - 20\cos^3\theta\sin^3\theta + 6\cos\theta\sin^5\theta$$

(b) Solve the equation

$x^3 - 6x^2 + 11x - 6 = 0$, the roots being in Arithmetic Progression (A.P.).

(c) If α, β, γ be the roots of the equation

$x^3 + px^2 + qx + r = 0$, $r \neq 0$, find the value of

(i) $\sum \alpha^2\beta$, (ii) $\sum \left(\frac{\alpha^2}{\beta\gamma}\right)$.

PART B (Calculus)

SECTION IV

4. (a) Show that $f(x) = |x + 1|$, $x \in R$ is continuous at $x = -1$.

(b) If $y = \cos(m \sin^{-1}x)$ show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_n = 0.$$

(c) If $u = \cos^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0.$$

P.T.O.

SECTION V

5. (a) Show that the tangent at any point of the curve

$$x = ae^{\theta}(\sin\theta - \cos\theta)$$

$$y = ae^{\theta}(\sin\theta + \cos\theta)$$

is at a distance ae^{θ} from origin.

(b) Find the asymptotes of the curve

$$(a + x)^2(b^2 + x^2) = x^2y^2$$

(c) Trace the curve $y^2x^2 = x^2 - 1$.

SECTION VI

6. (a) Evaluate

(i) $\int \frac{x^2\,dx}{(x^2+a^2)(x^2+b^2)}$ (ii) $\int \frac{4 + 5\tan x}{3 + 2\tan x}\,dx$

(b) Find the area between the curve

$$y^2(a + x) = (a - x)^2$$

and its asymptote.

(c) Find the surface of sphère of radius a.