[This question paper contains 4 printed pages.]

4491-A *Your Roll No.* .....

B.A. / II **AS**

(T)

MATHEMATICS – Paper II

(Geometry, Vector Calculus and Differential Equations)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

All *questions are compulsory.*
Attempt any **two** *parts from each Section.*

SECTION I

1. (a) Find the equation of the circle which intersects the circles

$$x^2 + y^2 - 6y + 1 = 0$$

$$x^2 + y^2 - 4y + 1 = 0$$

orthogonally and touches the line

$$3x + 4y + 5 = 0$$

(b) Show that the tangent at one extremity of a focal chord of a parabola is parallel to the normal at the other end.

P.T.O.

(c) Show that the locus of the middle points of a system of parallel chords of a parabola is a line parallel to the axis of the parabola. (6,6)

SECTION II

2. (a) Prove that the line $x\cos\alpha + y\sin\alpha = p$ is a tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ if } p^2 = a^2\cos^2\alpha + b^2\sin^2\alpha$$

(b) The perpendiculars from the centre upon the tangent at any point of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meets the tangent at p. Find the locus of p.

(c) Trace the conic

$$x^2 + 2xy + y^2 - 2x - 1 = 0 \qquad (6,6)$$

SECTION III

3. (a) Obtain the equation of the sphere having the circle

$$x^2 + y^2 + z^2 + 10y - 4z - 8 = 0,\ x + y + z = 3$$

as the great circle.

(b) Find the equation of the right circular cone with its vertex at the origin, axis along z-axis and semi-vertical angle α.

(c) Find the equation of the right circular cylinder of radius 2 whose axis is the line

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2} \qquad (6,6)$$

SECTION IV

4. (a) Find $\text{div}\vec{A}$ and $\text{curl}\vec{A}$ if

$$\vec{A} = xy^2z^2\,\hat{i} + \left(y^2 + z^2\right)\hat{j} + x^3z^2\,\hat{k}$$

(b) Show that $\vec{\nabla} \times (\vec{\nabla} \times \phi) = 0$.

(c) If $\vec{A} = t^2\,\hat{i} + 2t\,\hat{j} - (1+t)\hat{k}$

$$\vec{B} = (2t-1)\hat{i} - t^2\,\hat{j} + t\hat{k}$$

Find $\frac{d\vec{A}}{dt} \cdot \frac{d\vec{B}}{dt}$. (6½,6½)

SECTION V

5. (a) Solve $p^2 + 2py \cot x = y^2$.

(b) Solve $(D^3 + 3D^2 + 2D)y = x + 2$ where $D \equiv \frac{d}{dx}$.

(c) Show that e^x and e^{2x} are linearly independent solutions of

$$y'' - 3y' + 2y = 0 \qquad (6½,6½)$$

P.T.O.

SECTION VI

6. (a) Solve $\frac{dx}{x^2(y^2-z^2)} = \frac{dy}{y^2(z^2-x^2)} = \frac{dz}{z^2(x^2-y^2)}$.

(b) Solve, using the method of undetermined co-efficients,

$$(D^2 + 9)y = e^x + 2.$$

(c) Solve

$$(yz + xyz)dx + (zx + xyz)dy + (xy + xyz)dz = 0$$

(6½,6½)