[This question paper contains 6 printed pages.]

Your Roll No. ....

4492-A **AS**

B.A.(Pass)/III

MATHEMATICS–Paper-III (*i*)

(Real Analysis and Mechanics)

Time : 3 *Hours* *Maximum Marks* : 100

(*Write your Roll No. on the top immediately on receipt of this question paper.*)

Note : The maximum marks printed on the question paper are applicable for the students of Category 'B'. These marks will, however, be scaled down proportionately in respect of the students of regular colleges. Category 'A' at the time of posting of awards for compilation of result.

Note : Use separate answer-sheets for **Part A (Real Analysis)** and **Part B (Mechanics).**

Attempt any *two* parts from each Section.

All questions carry equal marks.

Part A (Real Analysis)

Section I

1. (*a*) Define neighbourhood of a point. Show that every open interval is a neighbourhood of all of its points.

(*b*) Show that the set of integers has no limit points in $\mathbb{R}$.

(*c*) Show that a function $f : \mathbb{R} \to \mathbb{R}$ is continuous at a point $a \in \mathbb{R}$ if and only if for any sequence $<a_n>$ in $\mathbb{R}$ with $a_n \to a$ we have $f(a_n) \to f(a)$.

(*d*) Show, with the help of an example, that a continuous function need not be uniformly continuous.

Section II

2. (*a*) Show that a bounded monotone sequence is convergent.

(*b*) Find $\limsup_{n \to \infty} a_n$ and $\liminf_{n \to \infty} a_n$, when :

(*i*) $a_n = (-1)^n + \frac{1}{n}$

(ii) $a_n = \frac{(-1)^n}{n}$.

(*c*) Test for the convergence :

(*i*) $\sum_{n=1}^{\infty} \frac{\sin nx}{n^2}, x \in \mathbb{R}$

(*ii*) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^p}, \ p \in \mathbb{R}$.

(*d*) State and prove Leibnitz Test for alternating series.

Section III

3. (*a*) State and prove Taylor's Theorem.

(*b*) Prove that for $0 < |x| \le \frac{\pi}{2}$,

$$\frac{2}{\pi} \le \frac{\sin x}{x} < 1.$$

(*c*) Examine for extrema :

(*i*) $\sin x + \cos x$,

(*ii*) $\left(\frac{1}{x}\right)^x$.

(*d*) Evaluate :

(*i*) $\lim_{x \to 0} \frac{(1+x)^{1/x} - e}{x^2}$

(*ii*) $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$.

Part B (Mechanics)

Section IV

4. (*a*) Three like parallel forces P, Q, R act at the vertices of the triangle ABC. If their resultant passes through

the circumcentre in all cases, whatever be the common direction of the forces, show that :

$$\frac{P}{\sin 2A} = \frac{Q}{\sin 2B} = \frac{R}{\sin 2C}.$$

(*b*) A uniform square lamina rests in equilibrium in a vertical plane under gravity with two of its sides in contact with smooth pegs in the same horizontal line at a distance C apart. Show that the angle θ made by a side of the square with the horizontal in a non-symmetrical position of equilibrium is given by

$$C(\sin\theta + \cos\theta) = a,$$

$2a$ being the length of a side of the square.

(*c*) A ladder whose centre of gravity divides it into two portions of lengths a and b rests with one end on a rough horizontal floor and the other end against a rough vertical wall. If the coefficient of friction at the floor and the wall be respectively μ and μ'. Show that the inclination of the ladder to the floor, when equilibrium is limiting is given by

$$\tan^{-1} \frac{a - b\mu\mu'}{\mu(a+b)}$$

(d) A piece of wire of given length is bent into the form of a quadrant of a circle and its two bounding radii. Find the centre of gravity of the whole.

Section V

5. (a) A particle moves in a straight line with uniform acceleration and its distances from the origin O on the line (not necessarily the position at $t = 0$) at time t_1, t_2, t_3 are d_1, d_2, d_3 respectively. Prove that if t_1, t_2, t_3 form an A.P., where common difference is d, and d_1, d_2, d_3 are in G.P., then the acceleration is

$$\frac{\left(\sqrt{d_1} - \sqrt{d_3}\right)^2}{d^2}.$$

(b) If α, β are the two possible directions to hit a given point (a, b), show that

$$\tan(\alpha + \beta) = -\frac{a}{b}.$$

(c) A particle of mass m is placed on a horizontal board, which is made to execute vertical simple harmonic oscillations of period T and amplitude a. Show that the particle does not lose contact with the board at any time if $a < \left(\dfrac{gT^2}{4\pi^2}\right)$.

(*d*) A heavy particle rests on top of a smooth fixed sphere. If it is slightly displaced, find the angular distance from the top at which it leaves the surface.

Section VI

6. (*a*) A parallelogram has the highest angular point in the surface of a liquid and one diagonal horizontal. Show that the depth of its centre of pressure is $\frac{7}{12}$ of the depth of the lowest point.

(*b*) Prove that in a fluid at rest under gravity horizontal planes are surfaces of equal density.

(*c*) A rectangular area is immersed in a heavy liquid with two sides horizontal, and is divided by horizontal lines into strips on which the total thrusts are equal. Prove that if a, b, c are the breadths of three consecutive strips,

$$a\,(a + b)\,(b - c) = c\,(b + c)\,(a - b).$$