This question paper contains 4+2 printed pages]

Your Roll No........................

5433

B.A. Prog./I **J**

(A)

MATHEMATICS

Paper I

(Algebra and Calculus)

(New Course : Admissions of 2004 and onwards)

Time : 3 *Hours* *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :— The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

All Sections are compulsory and have equal marks.

Attempt any *two* parts from each Section.

P.T.O.

Section I

1. (a) Define subspace of a vector space over a field. Prove that a non-empty subset W of a vector space V over F is a subspace of V if and only if $\alpha w_1 + \beta w_2 \in W \ \forall \ \alpha, \beta \in F$ and $\forall \ w_1, w_2 \in W$.

(b) Find the rank of the matrix :

$$\begin{pmatrix} 1 & 1 & 2 & 3 \\ 1 & 3 & 0 & 3 \\ 1 & -2 & -3 & -3 \\ 1 & 1 & 2 & 3 \end{pmatrix}$$

by reducing it to normal form by means of elementary transformations.

(c) Find the characteristic equation of the matrix :

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 0 & -1 \end{pmatrix}$$

and hence compute its cube.

Section II

2. (*a*) If

$$\sin\theta + \sin\phi + \sin\psi = 0 = \cos\theta + \cos\phi + \cos\psi,$$

show that

$$\sin 3\theta + \sin 3\phi + \sin 3\psi = 3\sin(\theta + \phi + \psi)$$

and $\cos 3\theta + \cos 3\phi + \cos 3\psi = 3\cos(\theta + \phi + \psi)$.

(*b*) Solve the equation :

$$z^4 + 4z^2 + 16 = 0.$$

(*c*) If α, β, γ are the roots of the equation :

$$x^3 + px^2 + qx + r = 0$$

find the values of :

(1) $\sum \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$

(2) $\sum \alpha^3$.

Section III

3. (a) Find the value of a if the function f given by :

$$f(x)=\begin{cases}2x-1 & , \quad x<2 \\ a & , \quad x=2 \\ x+1 & , \quad x>2\end{cases}$$

is continuous at $x = 2$.

(b) If

$$\cos^{-1}(y/b) = \log (x/n)^n,$$

prove that

$$x^2y_{n+2} + (2n+1)\, xy_{n+1} + 2n^2y_n = 0.$$

(c) Verify Euler's theorem for :

$$z = \sin^{-1}(x/y) + \tan^{-1}(y/x).$$

Section IV

4. (a) Prove that the sum of the intercepts on the coordinate axes of any tangent to the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ is constant.

(b) Show that the radius of curvature of the curve $r^m = a^m \cos m\theta$ is :

$$\frac{a^m}{(m+1)\,r^{m-1}}.$$

(c) Trace the curve :

$$y^2(a + x) = x^2(3a - x).$$

Section V

5. (a) Using the function $f(x) = e^{-x} - \cos x$, show that between any two roots of $e^x \cos x = 1$, there exists at least one root of $e^x \sin x - 1 = 0$.

(b) Find the maximum and minimum values for the function :

$$f(x) = x^4 + 4x^3 - 2x^2 - 12x + 7.$$

(c) Evaluate :

$$\lim_{x \to 0} \frac{e^x \sin x - x - x^2}{x^2 + x \log(1 - x)}.$$

Section VI

6. (a) If

$$u_n = \int_0^{\pi/2} x^n \sin x \, dx, (n > 1)$$

then prove that :

$$u_n + n(n-1)\, u_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}.$$

Deduce the value of u_5.

(b) Find the area of the smaller portion enclosed by the curves :

$$x^2 + y^2 = 9; \; y^2 = 8x.$$

(c) Prove that the whole length of the curve :

$$x^2(a^2 - x^2) = 8a^2y^2$$

is $\pi a\sqrt{2}$.